TET TROUBLE

Show that is it impossible to have a tetrahedron whose six edges have lengths 10, 20, 30, 40, 50 and 60 units.

Is it possible for a tetrahedron to have edges of lengths 10, 20, 25, 45, 50 and 60 units?

Can you write general rules for someone else to use to check whether a given six lengths could form the edges of a tetrahedron?

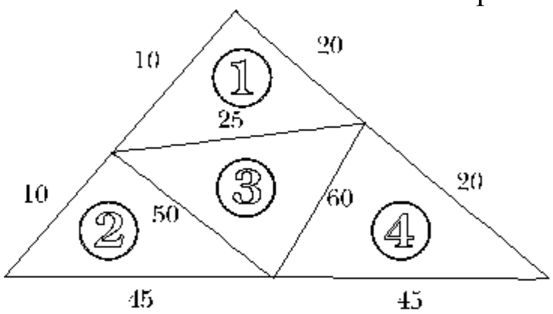
SOLUTION

It is impossible to have a tetrahedron which has edge lengths of 10, 20, 30, 40, 50 and 60 units.
This is because to produce a triangle two sides have to be bigger (when added together) than the third. This is called the ***TRIANGLE INEQUALITY.***
So this tetrahedron won't work because the 10 is too small to make a triangle. e.g. 10, 20, 30

- $20 + 30 > 10$ (OK)
- $10 + 30 > 20$ (OK)
- but $10 + 20 = 30$ so the triangle becomes a straight line.

10 will not go with any other two numbers in the above list.

It is possible to make a tetrahedron which has lengths 10, 20, 25, 45, 50 and 60 units. The reason is that the numbers are big and small enough to fit the triangle inequality (the sum of two sides bigger than the third). Here is a net of this tetrahedron to explain this answer (not to scale!)



Triangle 1 $10 + 25 > 20$
$10 + 20 > 25$
$20 + 25 > 10$

Triangle 2 $10 + 50 > 45$
$10 + 45 > 50$
$50 + 45 > 10$

Triangle 3 $50 + 60 > 25$
$25 + 60 > 50$
$25 + 50 > 60$

Triangle 4 $60 + 45 > 20$
$20 + 60 > 45$
$20 + 45 > 60$

All four faces satisfy the triangle inequality, so it can make a tetrahedron.

The general rule for it to be possible to make a tetrahedron with edge lengths given by a set of 6 numbers is that there are 4 triplets chosen from these numbers that satisfy the triangle inequality, for example {a, b, c}, {a, d, e}, {b, e, f} and { c, f, d}.

Triangle 1 $a + b > c$
$a + c > b$
$b + c > a$

Triangle 2 $a + d > e$
$a + e > d$
$d + e > a$

Triangle 3 $b + e > f$
$b + f > e$
$e + f > b$

Triangle 4 $c + f > d$
$c + d > f$
$d + f > c$

NOTES FOR TEACHERS

Why do this activity?

This activity offers an excellent opportunity for learners to practise visualisation and apply an idea normally only used in 2D geometry to a 3D case. Learners will have to consider carefully how to communicate their methods for testing combinations and be sure that they have considered all possibilities.

Possible approach

Ask learners to pay special attention to what you are doing and write three lengths on the board (for example 3 units, 6 units, 7units) and draw a triangle with sides of corresponding lengths as accurately as possible. Do it again with three more lengths.

Then again, but instead of drawing the triangle put 3 question marks. After some thinking time, encourage a member of the group to come up and draw the triangle.

Finally, list three lengths that will not work followed by a question mark. Use straws of these lengths to try to make a triangle. After time has been taken to realise the impossibility, the class should discuss why this is the case.

Now pose the problem. Working in small groups the challenge will be to use systematic approaches as well as applying the triangle inequality.

Take opportunities to pull together different ideas for recording, including the use of nets and working systematically.

Key questions

- How do you know you have tried all possibilities?
- Is it possible to construct more than one tetrahedron?
- Can you find six lengths which will give more than one tetrahedron?

Possible extension

The problem Triangles to Tetrahedra https://aiminghigh.aimssec.ac.za/grades-9-to-11-triangles-to-tetrahedra/ requires learners to work systematically to generate all possible tetrahedra from four particular triangles.

Possible support

Use construction straws of equivalent lengths to make (or fail to make) triangles and tetrahedra. Alternatively, draw nets and cut them out to see if they 'work'.