AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE
TEACHER NETWORK

SQUARE HOLE

If you take the side length of the equilateral triangles as the unit for length, what would be the exact size of the hole?

Alternatively, if the area of the equilateral triangles is taken as the unit for area, what size is the hole then?

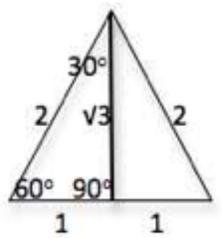
SOLUTION

The design given is made up of 4 rectangles with 2 equilateral triangles and 2 isosceles triangles in them. Each rectangle can be split into two 30^o–60^o right-angled triangles and two isosceles triangles.

The diagram below shows an equilateral triangle with sides twice as long.

From the special triangle, if the edge length is 1 unit then the height is $\sqrt{3}/2$ and area $\sqrt{3}/4$.

The large outer square has edge $1 + \sqrt{3}$ and the **small inner square has edge** $\mathbf{(1 + \sqrt{3}) - 2 = \sqrt{3} - 1}$.



The area of the small square (the hole) is then $(\sqrt{3} - 1)^2 = 3 - 2\sqrt{3} + 1 = \mathbf{4 - 2\sqrt{3}}$**.**

If the area of the equilateral triangle is 1 square unit then **the area scale factor is** $\mathbf{4/\sqrt{3}}$ so **the area of the hole is** $(4 - 2\sqrt{3}) \times 4/\sqrt{3} = \mathbf{16\sqrt{3}/3 - 8}$**.**

Alternatively, if the area of the equilateral triangle is 1 square unit and the height is h, then $h^2/\sqrt{3} = 1$ so $h^2 = \sqrt{3}$.

The large square has edge length $2h + 2h/\sqrt{3}$

The hole has edge length $(2h + 2h\sqrt{3}) - 4h/\sqrt{3} = 2h(\sqrt{3} - 1)/\sqrt{3}$.

The area of the hole is $[2h(\sqrt{3} - 1)/\sqrt{3}]^2 = 4h^2(4 - 2\sqrt{3})/3 = \mathbf{16\sqrt{3}/3 - 8}$

NOTES FOR TEACHERS

Why do this activity?

The equilateral triangle with a perpendicular from a vertex to the base (line of symmetry) splitting it into two congruent 30–60–90 triangles is the key to remembering some important geometric and trigonometric facts. If learners are familiar with it they will save a lot of time as they will be able to recall those facts easily.

By asking for the ***exact*** size, this problem forces consideration of the edge lengths in surd form, and gives practice in calculating with surds.

Changing from lengths to areas emphasises that problem solvers may have to choose their own unit of length to solve a problem and by choosing the unit they can sometimes make the problem easier to solve.

Intended Learning Objectives (Grade 11)

- To be able to add, subtract, multiply and divide simple surds and simplify expressions involving surds.
- To reinforce visualisation and memory of the properties of the equilateral triangle and special angles.

Possible approach

You could start the lesson by asking “what can you tell me about equilateral triangles?’ and perhaps ask a learner to come and draw one on the board. Keep asking “can you tell me anything else?” and mark all the properties suggested on the diagram. You may want to revise their recall of the trig ratios although they are not used in this activity.

Then hand out photocopies of the worksheet or draw the diagram and write the question on the board. Ask the learners to describe what they see in the picture and make a list of what they notice.

Then ask the learners to work individually on the activity, and after 5 minutes to work with a partner. When most pairs have an answer tell the class to work in fours and to compare their answers and explain to the other pair how they found them. In this way learners will correct some of their own errors and help each other to understand what to do.

In the whole class plenary you might, if time, ask a pair of learners to come to the front and explain their work, one learner writing on the board while the other one gives the explanation.

It is important for the teacher to summarise, at the end of lessons, the main ideas and methods that have been used and, in this case, to review the methods of calculation with surds.

Key questions

- What shapes can you see in the picture?
- Is there a connection between the lengths of the edges of the rectangles and the edges of the equilateral triangles?
- What is the area of the yellow equilateral triangle in terms of its edge length ?
- What is the relationship between the area of the yellow triangle and the area of the purple triangle? Why?
- How big is the outer square? How do you know?
- If you know the edge length of the outer square how can you find the edge length of the inner square?

Possible extension

The yellow triangles in this diagram are equilateral.

- Can you make a regular hexagon using only yellow triangles ?
- Can you make a regular hexagon using only purple triangles ?
- Can you make a regular hexagon from yellow triangles that is the same size as a regular hexagon made from purple triangles ?

Did that tell you something about yellow and purple triangles, about how they relate to each other ?

- Can you make a larger equilateral triangle from yellow triangles ?
- Can you make an equilateral triangle from purple triangles ?
- Can you make an equilateral triangle from yellow triangles that is the same size as an equilateral triangle made from purple triangles?

Possible support

Some learners will benefit from playing with cut out triangles to make different patterns. This is a valuable basis for other mathematical ideas to be built on.

There are a number of activities that can provide valuable experiences for learners working on this activity.

Drawing first the equilateral triangle using only a straight edge and compasses, and then creating the isosceles triangle in the same way, will give a strong sense for the symmetry of each triangle and the relationship between them.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA and to Years 4 to 12 in the UK.				
	Lower Primary or Foundation Phase	Upper Primary	Lower Secondary	Upper Secondary
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6