SCHOOL PHOTOS

A school photographer is taking photographs.

She has to arrange the people in two rows one behind the other in order of height.

Each person at the back must be taller than the person directly in front of them.

Along the rows the heights must increase from left to right.

How many different arrangements are there for the photo if there are four people in the group?

What about groups of six, or eight?

In how many ways can the basketball team of 10 players be arranged like this for the photo to be taken?

If you want a big challenge you could look for the number of arrangements for twelve people, or any given even number.

HELP

The best way to do this is to work in groups and actually stand in the positions for the photo. To record your results number the people 1 to 4 with 1 the smallest and 4 the tallest. Then record the arrangements such as: 3 4   or   2 4   etc.

1 2   1 3

When you have found all the arrangements for 4 people do the same for 6 people.

NEXT

To find the answer for 8 people make 8 cards numbered 1 to 8 and arrange them in two rows of 4 according to the rules given. How many ways can you do this?

There is an obvious generalisation to taking a photo of 10, 12, 14, ... and then of any even number of people.

Notes for Teachers

SOLUTION

Label the people by the numbers 1, 2, 3, 4... in ascending order of height. As each person at the back must be taller than the person directly in front of them and, along the rows, the heights must increase from left to right the 1 must be in front on the left and the tallest must be at the back on the right. The results for 2, 4, 6, and 8 people are in the following table.

Number of people	Number of arrangements					
2	1	2 1				
4	2	3-4 1-2	2-4 1-3			
6	5	4-5-6 1-2-3	3-5-6 1-2-4	3-4-6 1-2-5	2-5-6 1-3-4	2-4-6 1-3-5
8	14	5-6-7-8 1-2-3-4	4-6-7-8 1-2-3-5	4-5-7-8 1-2-3-6	4-5-6-8 1-2-3-7	
		3-6-7-8 1-2-4-5	3-5-7-8 1-2-4-6	3-5-6-8 1-2-4-7		
		3-4-7-8 1-2-5-6	3-4-6-8 1-2-5-7			
		2-6-7-8 1-3-4-5	2-5-7-8 1-3-4-6	2-5-6-8 1-3-4-7		
		2-4-7-8 1-3-5-6	2-4-6-8 1-3-5-7			

Many people think that because the sequence 1, 2, 5, 14 ? goes up in powers of 3 (with differences 1, 3 and 9) the next difference will be 3 cubed to give the next number $14 + 27 = 41$. Maths is full of patterns but, when you think you spot a pattern you have to prove it always works. If you count the arrangements for 10 people the answer is 42 and not 41.

These are the 42 arrangements for a team of 10

6-7-8-9-10 1-2-3-4-5	5-7-8-9-10 1-2-3-4-6	5-6-8-9-10 1-2-3-4-7	5-6-7-9-10 1-2-3-4-8	5-6-7-8-10 1-2-3-4-9
4-7-8-9-10 1-2-3-5-6	4-6-8-9-10 1-2-3-5-7	4-6-7-9-10 1-2-3-5-8	4-6-7-8-10 1-2-3-5-9	
4-5-8-9-10 1-2-3-6-7	4-5-7-9-10 1-2-3-6-8	4-5-7-8-10 1-2-3-6-9		
4-5-6-9-10 1-2-3-7-8	4-5-6-8-10 1-2-3-7-9			
3-7-8-9-10 1-2-4-5-6	3-6-8-9-10 1-2-4-5-7	3-6-7-9-10 1-2-4-5-8	3-6-7-8-10 1-2-4-5-9	

3-5-8-9-10 1-2-4-6-7	3-5-7-9-10 1-2-4-6-8	3-5-7-8-10 1-2-4-6-9	
3-5-6-9-10 1-2-4-7-8	3-5-6-8-10 1-2-4-7-9		
3-4-8-9-10 1-2-5-6-7	3-4-7-9-10 1-2-5-6-8	3-4-7-8-10 1-2-5-6-9	
3-4-6-9-10 1-2-5-7-8	3-4-6-8-10 1-2-5-7-9		
2-7-8-9-10 1-3-4-5-6	2-6-8-9-10 1-3-4-5-7	2-6-7-9-10 1-3-4-5-8	2-6-7-8-10 1-3-4-5-9
2-5-8-9-10 1-3-4-6-7	2-5-7-9-10 1-3-4-6-8	2-5-7-8-10 1-3-4-6-9	
2-5-6-9-10 1-3-4-7-8	2-5-6-8-10 1-3-4-7-9		
2-4-8-9-10 1-3-5-6-7	2-4-7-9-10 1-3-5-6-8	2-4-7-8-10 1-3-5-6-9	
2-4-6-9-10 1-3-5-7-8	2-4-6-8-10 1-3-5-7-9		

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

> You have three cards P, Q and R. One arrangement is QPR. How many arrangements can you make altogether?
>
> A. 3   B. 5   C. 6   D. 4

The correct answer is C. The arrangements are:
PQR, PRQ, QPR, QRP, RPQ, RQP

https://diagnosticquestions.com

Why do this activity?
This 'people' activity gets small groups physically involved in learning by action (kinaesthetic learning). It is very suitable for upper primary as well as for older students.

Learning objectives
In doing this activity students will have an opportunity to work systematically to find all possible arrangements.

Generic competences
In doing this activity students will have an opportunity to:
- **work in a team**:
  - collaborate and work with a partner or group
  - have empathy with others, listen to different points of view
  - develop leadership qualities;
- **communicate** in writing, speaking and listening:
  - exchange ideas, criticise, and present information and ideas to others
  - analyze, reason and record ideas effectively.

Suggestions for teaching
Depending on how long you can spend on this activity, and the age of the class, you can ask the class to find the number of arrangements for this type of group photo for 6 or 8 or 10 people.

Get four people of different heights to arrange themselves for the photo. How many ways can they do it? What about two people?

Then the class can try the problem in groups of six people. There will be a lot of discussion about the ways of recording the different arrangements and checking that they have found them all.

If there is time the class can look for arrangements for 8 people and then for 10 people.

Having found the number of arrangements for 2, 4, 6 and 8 people there appears to be an obvious pattern that suggests the number of arrangements for 10 people. But then counting the possibilities leads to a surprise. What looked like an obvious pattern does not work for 10 people.

This is a low entry point, high ceiling activity. To identify the pattern and find a formula for the sequence that arises is very challenging. This is the sequence of Catalan numbers and high flyers may like to do some research on the internet to find out about these numbers. However you need to know about binary coefficients to be able to interpret the formulae.

Key questions
- You have the 4 people in the front row, how many ways can you put someone in the 5th place?
- Can you find a system for finding all the front row arrangements following the height order rules?

Follow up
One step two steps https://aiminghigh.aimssec.ac.za/years-7-10-one-step-two-steps/