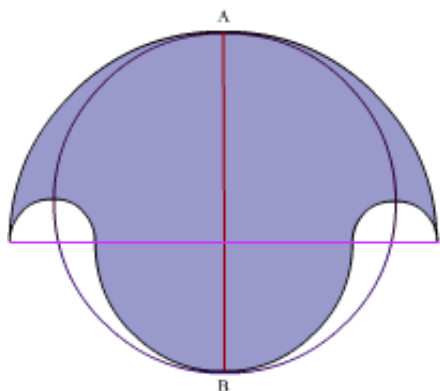


SALINON



This shape, which is constructed using four semi-circles, is called a Salinon.

If the radius of the larger blue semicircle is a and the radius of the smaller blue semicircle is b , what are the radii of the smallest semicircles?

What is the relationship between the area of the shaded region and the area of the circle on AB as diameter?

SOLUTION

Rather surprisingly perhaps the two areas are always the same however the radii of the semicircles are chosen.

The smallest semicircles have radius $= \frac{1}{2}(a - b)$ and area $\frac{1}{2}\pi[\frac{1}{2}(a - b)^2]$

$AB = (a + b)$ so the area of the circle on AB as diameter is $\frac{1}{4}\pi(a + b)^2$

And the area of the shaded region is $\frac{1}{2}\pi a^2 - \frac{1}{4}\pi(a - b)^2 + \frac{1}{2}\pi b^2 = \frac{1}{4}\pi(a + b)^2$

Alternatively if you label AB as $2R$ and the radii of the smallest semicircles as r then the radius of the largest semicircle is $R + r$ and the radius of the shaded semicircle below the horizontal line is $R - r$.

The circle on AB as diameter has area πR^2 .

The shaded area $= \frac{1}{2}\pi(R + r)^2 - 2(\frac{1}{2}\pi r^2) + \frac{1}{2}\pi(R - r)^2 = \frac{1}{2}\pi(R^2 + 2Rr + r^2 - 2r^2 + R^2 - 2Rr + r^2) = \pi R^2$

NOTES FOR TEACHERS

Why do this activity?

This activity is an exercise in finding the areas of circles and it gives practice in geometrical reasoning and manipulating algebraic expressions.

Intended Learning Objectives (Grades 9 and 10)

To review and practice problem solving related to calculating areas of circles in composite and complex figures.

Possible approach

Start with a class discussion about the symmetry in the diagram, the radii of the 4 semi-circles and the circle on AB as diameter.

When the students have understood that there are only two unknowns involved, and the other radii can all be expressed in terms of the two unknowns, then they are ready to write down the areas and answer the question which they could do individually.

Dynamic geometry software could be a useful tool to aid investigation but this is not essential.

Key questions

If you **add** the radius of the smaller blue semicircle to the radius of the larger blue semicircle what do you get?

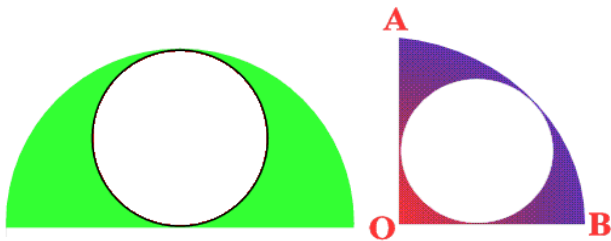
Can you see from the diagram how you could find the diameter of the smallest semicircle?

If you **subtract** the radius of the smaller blue semicircle from the radius of the larger blue semicircle what do you get?

What areas do you have to add and subtract to get the blue shaded area?

How would you simplify that algebraic expression?

Possible extension



Compare the areas of the semicircle and the circle inside it.

Compare the areas of quarter circle and the circle inside it.

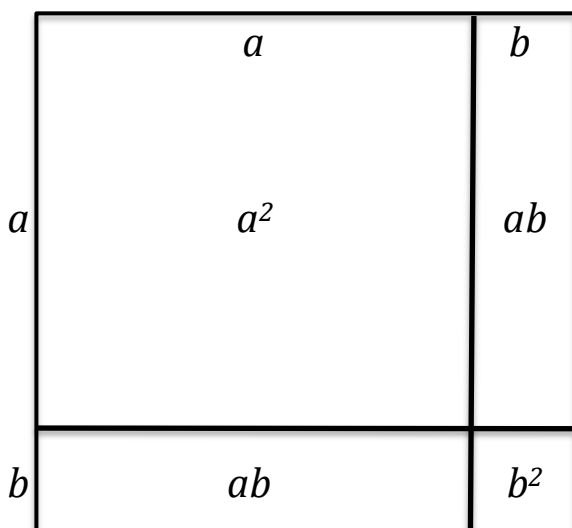
What about the areas if you change the angle AOB?

Learners could make up their own problems of this type.

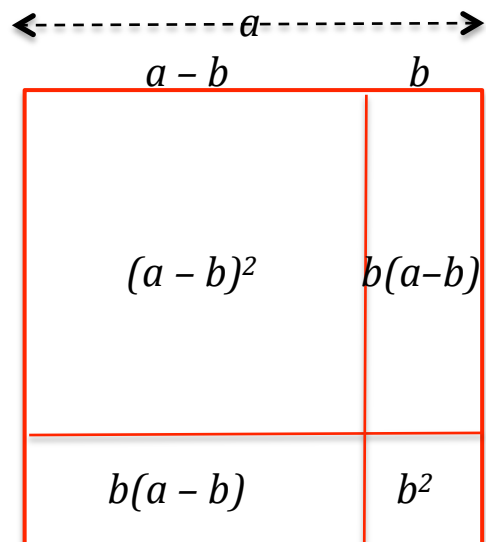
Possible support

Learners may need to help each other to understand why the radii of the semicircles are a , b and $\frac{1}{2}(a - b)$ and $AB = \frac{1}{2}(a + b)$.

Learners may find the diagrams below help them when they come to simplify the formulas in this activity.



$$(a + b)^2 = a^2 + 2ab + b^2$$



$$\begin{aligned}(a - b)^2 &= a^2 - 2b(a - b) - b^2 \\ &= a^2 - 2ab + 2b^2 - b^2 \\ &= a^2 - 2ab + b^2\end{aligned}$$