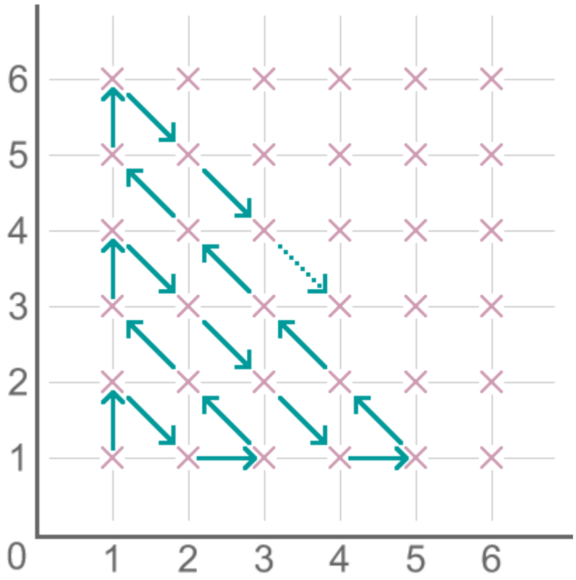
AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE
TEACHER NETWORK

# ROUTE TO INFINITY




Look at the route the arrows follow in this diagram. Look away from the screen and try to describe their path.

Will the route pass through the point (18,17)?

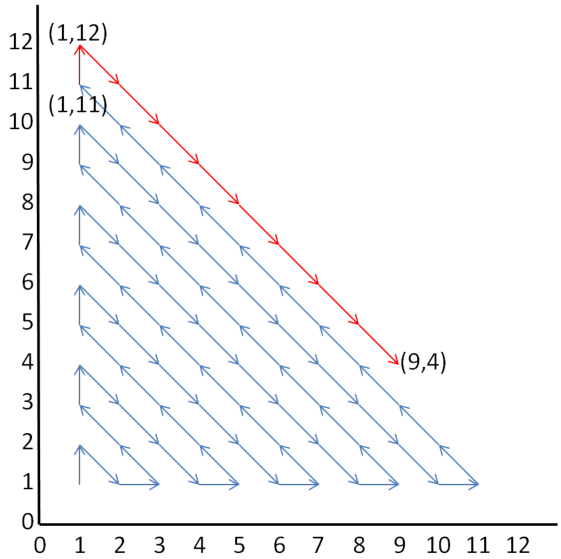
If so, which point will be visited next? Explain how you found out.

How many points does the route pass through before it reaches the point (9,4)?

Explain how you found out.

If this diagram is extended to the whole first quadrant, and the route continues in the same way, what can you say about the lattice points it goes through? (A lattice point is a point with integer coordinates.)

# SOLUTION




The route starts at (1, 1) and goes up to (1, 2). It follows a continuous path in the first quadrant, through all the lattice points (points with whole number coordinates), along each line x + y = constant going down for odd c and up for even c.

From all the points (1, 1) (3, 3), … , (2n-1, 2n-1)… the route goes up to (1, 2), (3, 4), … , (2n-1, 2n) … and from all the points (2, 2) (4, 4), … , (2n, 2n)… the route goes across to (2, 3), (4, 5), … , (2n, 2n+1).

The point (18, 17) is on x + y = 35 so the route is going down through that point and will visit (19, 16 next).

On each line segment of x + y = c the route goes through (c-1) points so the totals as it reaches the end of each segment are the triangle numbers. Before (9, 4) it will have visited (1 + 2 + 3 + 4 … 11) + 8 = 66 + 8 = 74 points.

If this diagram is extended to the whole first quadrant, and the route continues in the same way then the route passes through all the infinitely many lattice points in order.

Each of these points represents a rational number, the coordinates (a, b) representing the rational number a/b. All the rational numbers are included in a clearly defined order so they are countable from 1 to infinity as they are reached on the route.

# NOTES FOR TEACHERS

## Why do this activity?

This activity offers a good opportunity for learners to visualise and discuss patterns, to develop their fluency with coordinates, to solve problems and to find convincing explanations and proofs for their findings. Describing the coordinate pattern provides the teacher with a way to help learners to extend their mathematical vocabulary and language fluency. The activity leads to thinking about infinity and it makes a connection with the idea of putting objects in order and counting them, and also with triangle numbers.

## Intended Learning Objectives (Grade )

To review representation of points and lines in the Cartesian co-ordinate system and the properties of lines through given points. To be able to investigate and extend numeric and geometric patterns looking for relationships between numbers including patterns represented in physical or diagram form and represented algebraically. To be able to describe and justify the general rules for observed relationships between numbers in learners' own words or in algebraic language.

## Possible approach

You might like to download the NRICH poster.

Start by saying "Have a look at this image. In a moment I'm going to remove it, and I want you to be able to describe the route that the arrows take to your partner."
Give learners a short while to look at the image, then remove it.

"Without using paper or pencil, can you describe the route to each other?"
Once they have done this, show them the image again to check that what they have described is indeed what they saw.

"I'd like one person in each pair to turn their back to the screen and list the coordinates in the order in which they're visited, and your partner to look at the screen and check. When you make a mistake, swap over. See how far you can get."

Once learners have spent some time listing the coordinates, bring the class together.
"I wonder if you can work out where the route will take you after visiting the point (18,17)? Spend a short while thinking about it on your own, then discuss it with your partner, and together develop a convincing explanation for your answer to share with the class."

As learners are working, if they get stuck you could offer the following hint:
"What do you notice about the coordinates of the points visited when the arrows are sloping upwards/downwards?"

Learners' explanations are likely to refer to specific examples on the visible grid. It is important to insist on clearly justified arguments that refer to the generality - a key question to ask is "How do you know it will **always** happen?".

Finally, introduce the last question: "I wonder if you can work out how many points the route will pass through before reaching (9,4)? Again, you may want to start by working on your own before discussing it with your partner, and then developing a convincing explanation to share with the class."

While pairs are talking, circulate and eavesdrop on discussions, correcting any misconceptions and making a mental note of any learners who have particularly clear explanations.

Bring the class together and invite those learners with interesting or elegant strategies to present their ideas to the rest of the class.

## Possible extension

Learners could also work on https://aiminghigh.aimssec.ac.za/grade-10-to-12-coordinate-patterns/

Challenge students to design a route that will cover every grid point on an infinite coordinate grid in all 4 quadrants, and to create some questions (and answers), like those above, to go with their design. They could then swap with a partner.

The thinking involved in this problem could lead onto some investigation into countable infinity. This article by Katherine Korner would make a good starting point.

## Possible support

You may like to work with the whole class in a large open space to review ideas of coordinates through a **people maths activity**. Plan for each learner to stand on a lattice point and to be physically involved. Mark out an x-axis and a y-axis on the ground. Give each learner a slip of paper with the coordinates of their own lattice point. Then tell all the learners whose points are on the x-axis to go and stand on the line you have drawn. Make sure they are equally spaced out. Then tell the y-axis learners to go and stand on the y-axis. Then tell learners on the line $x = 1$ to go and stand in position, then the line $x = 2$, then $x = 3$ etc until all the learners are in position.

Now tell learners to raise their arms if they have y coordinates as 1. What do they notice about the line of learners? Now if they have y coordinates of 4 etc.

Now ask them to put up their hands if their coordinates add up to 3. What to they notice?
Where are the people whose coordinates add up to 6?

Now what about the line of learners whose coordinates are equal? Then the line of learners whose y coordinate is twice their x-coordinate? And so on…

By asking questions like this so that the learners are all participating, and they can look around and see the lines of hands up, you will be able to review with the class all that they have learned about coordinates and equations of lines.