Title: Right Time Grades 9 to 11

RIGHT TIME

The hour and minute hands of my clock are at right angles.

How many degrees does each hand move forward in one minute?

How long will it be before they are at right angles again?

Be careful, it will not be at half past three!

Thank you to NRICH for this problem.

Solution

The minute hand moves faster than the hour hand so, at 3 o'clock the angle between them is decreasing. In one minute the minute hand moves forward 360/60 = 6 degrees as there are 60 minutes in an hour. The hour hand moves forward 30 degrees in 1 hour so it moves 30/60 = ½ degree in one minute.

The angle between the hands changes by 5½ degrees in each minute so for example after 1 minute and 2 minutes the angle between the two hands is $84\frac{1}{2}^O$ and 79° and so on.

The hands coincide after 90/5.5 minutes and after that the minute hand is ahead of the hour hand.

The hands are at right angles after 180/5.5 = 32 8/11 minutes (between 3.30 and 3.35).

Alternatively, if the hands are at right angles after x minutes then 5½ x = 180 so x = 32 8/11

Notes for teachers

Why do this activity?
This problem solving activity relates to time and it gives practice in mathematical reasoning but it only requires knowledge about angles and time learned in primary school.

Possible approach
Take a clock into the classroom. Set the hands at 3 o'clock. Ask 'what is the angle between the hands?" Then ask the learners to watch the hands as you move the time on the clock forward. Stop at about 3.15. Ask "What do you notice?" The hands will have moved very close together. Continue moving time on the clock and ask the learners to tell you when the hands are at right angles again? This will be between 3.30 and 3.35.

Now ask the learners to work in pairs and to find out the EXACT time when the hands will be at right angles again. When several pairs have worked it out get them to come to the board in pairs and to explain to the class how they found the answer.

Then, to give everyone the opportunity to work successfully with these ideas, ask them to work out when the hands will next be at right angles to each other and give them time to do this. Ask different pairs of learners to explain this to the class.

Key questions
Does the angle between the hands of the clock stay the same as time changes?
How many degrees does the minute hand move forward in one minute?
How many degrees does the hour hand move forward in one hour? In one minute?
What will the angle between the hands be after 1 minute, 2 minutes, 3 minutes…?

Possible extension
Very able learners may be able to find the answer algebraically.
More challenges: How many times in a day will the hands be at right angles? How many times will they coincide?

Possible support
Learners could continue the sequence listing the angle between the hands at intervals of 1 minute
$84\frac{1}{2}^{\text{O}}$, 79°, $73\frac{1}{2}^{\text{o}}$, 67°, $61\frac{1}{2}^{\text{o}}$, 56°, …
Ask them how long it will take to get to 2°. Answer 16 minutes. Then what happens?