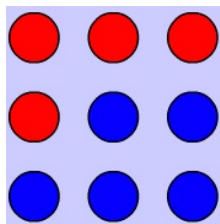


RED EVEN



(a) You have a 3 by 3 grid and you must mark the squares to make 4 red squares and 5 blue squares.

How many ways can you do this so that all the rows columns and diagonals have an even number of red squares?

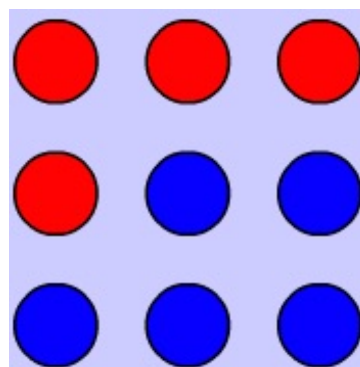
(b) Are there any additional solutions if only the rows and columns need to have an even number of red squares?

Two solutions are considered the same if one can be transformed to the other by rotating the square.

Help

Cut out counters from scrap card, or use bottle tops, so that you can move them around to make the different arrangements.

To record your results, use squared paper and 2 colours (or write R and B in the positions).



Extension

As an extension task find how many different arrangements you can make with 2 reds and 2 blues in a row.

Then choose for yourself a number of reds and blues (e.g. 1 red, 4 blues) and challenge a partner to see who can find the most arrangements in a line

NOTES FOR TEACHERS

SOLUTION

(a) There are only three places for the first red and only one leads to a solution:

- the middle,
- the middle of an edge or
- a corner

i. If the middle is red, then two more reds would be needed to go in corners to give both diagonals two reds. Making any other square red (the 4th red) will leave at least one row or column with a single red.

ii. If the middle of an edge is red then another red is needed for a corner, another to go on a diagonal and another to go along the side and there is still another row (or column) with a single red.

iii. The only solution is to have the 4 corners red.

R	B	R
B	B	B
R	B	R

R	B	R
B	B	B
R	B	R

R	R	B
R	R	B
B	B	B

R	R	B
B	B	B
R	R	B

(b) If the diagonals do not need to have an even number of reds then there are two more solutions.

Diagnostic Assessment This should take about 5–10 minutes.

- Write the question on the board, say to the class:
“Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D”.
- Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
- Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
- Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
- If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

The correct answer is D.

Sums in ascending order.

The columns start with the same number.

$1 + 5 + 9$ $2 + 4 + 9$ $3 + 4 + 8$ $4 + 5 + 6$
 $1 + 6 + 8$ $2 + 5 + 8$ $3 + 5 + 7$
 $2 + 6 + 7$

All learners will probably give correct answers because the question asks how many ways they can write these sums NOT how many ways can the sums be written. It is helpful to encourage learners by telling them that.

How many ways can you write 15 as the sum of 3 whole numbers from 1 to 9 without repetition?
 For example $1 + 5 + 9 = 15$

A. 3 ways B. 5 ways C. 6 ways D. 8 ways

Ask one learner to write his/her answers on the board. Then ask if anyone else has a different answer. Each time check that the ‘new’ sum is different from the other sums on the board. Keep encouraging the learners to find more solutions until the class has found all eight solutions. See <https://aiminghigh.aimssec.ac.za/fifteen-game/>

Why do this activity?

This is a practical activity and a simple investigation that gives learners the chance to develop mathematical thinking skills by trying out different possibilities and reasoning about the number of odd and even squares. The activity also involves talking about which solutions are the same because they are rotations of each other and the class can discuss reflection because the same pair are also reflections of each other.

Learning objectives

In doing this activity students will have an opportunity to deepen understanding of symmetry.

Generic competences

In doing this activity students will have an opportunity to:

- **think mathematically and flexibly**, reason logically and give explanations;
- **visualize** and develop the skill of interpreting and creating visual images;
- **persevere and work systematically** to investigate all possible cases.

Suggestions for teaching

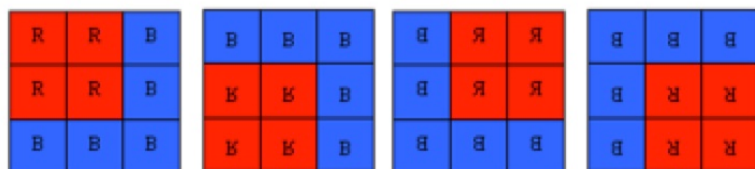
Start with the Diagnostic Quiz to involve learners in thinking about different arrangements of objects.

Draw a grid on the board and ask the learners to copy it. Write on the board:

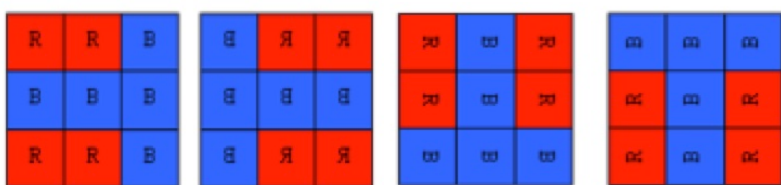
“you must mark the squares to make 4 red squares and 5 blue squares with an even number of reds in each row, each column and both diagonals”.

The learners can do this by writing R or B in each square, or by colouring the square, or using red and blue markers that can be moved around (counters or scraps of paper).

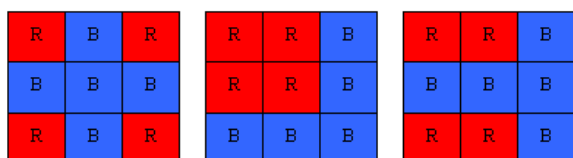
Give the learners 10 – 15 minutes to find some solutions. Then ask learners to suggest solutions and to draw each one on the board. For each solution suggested, ask learners to explain why that arrangement does or does not satisfy the given conditions. Do not erase any that are rejected but mark with a tick or cross.



Then ask which arrangements on the board are reflections or rotations of each other. Agree that you will only keep one of each type when you erase reflections and rotations. So all the top row are the same type and all the second row are the same type.



The class can narrow their choices to 3 cases.



Finally the learners should be able to count the numbers of reds in rows and columns and argue that they have considered all possibilities and the arrangement with the reds in the corners is the only solution.

Then ask the class if there is more than one solution if the diagonals do not need to have an even number of reds. Now let the learners talk about the other possibilities. Again ask the learners to explain their answers.

Key questions

- Which squares have to be red to make up the even numbers?
- Are those solutions the same or different?
- Show me two of the arrangements that are rotations of each other.
- What angle of rotation transforms that pattern to that one?
- What mirror line makes that pattern a reflection of that one?

Follow up

Another task involving symmetry in a 3by 3 square:

<https://aiminghigh.aimssec.ac.za/years-4-7-symmetry-challenge/>

Can you work out a strategy so that you never lose this game?

<https://aiminghigh.aimssec.ac.za/fifteen-game/>