



$a, a+1, a+2, a+3$

PAIR PRODUCTS

Choose four consecutive whole numbers, for example, 4, 5, 6 and 7.

Multiply the first and last numbers together (28).

Multiply the middle pair together (30).

Choose different sets of four consecutive whole numbers and do the same.

What do you notice about the difference between the two products each time?

Choose five consecutive whole numbers, for example, 3, 4, 5, 6 and 7.

Multiply the first and last numbers together (21).

Multiply the second and fourth numbers together (24).

Choose different sets of five consecutive whole numbers and do the same.

What do you notice now about the difference between the two products?

What happens when you take 6 or 7 or 8, or ... n consecutive whole numbers and compare the product of the first and last numbers with the product of the second and penultimate numbers?

Explain your findings.

HELP

For the example of the 4 consecutive whole numbers 4, 5, 6 and 7

$4 \times 7 = 28$ and $5 \times 6 = 30$ and the difference $30 - 28 = 2$.

Take another example: 6, 7, 8, 9 then $6 \times 9 = 54$ and $7 \times 8 = 56$ and the difference $56 - 54 = 2$.

Try some more examples. Is the difference always 2? Why.

What happens with 5 consecutive whole numbers?

NEXT

This problem only operated on the end numbers and the 'end but one' numbers. Generalise further by looking at other pairs within the sequence.

For example, if you have an odd number of consecutive numbers, what's the difference between the product of the end numbers and the square of the middle number?

Also try the differences of pair products for:

4 consecutive even numbers

4 consecutive odd numbers

5, 6, 7, 8, ... x consecutive even or odd numbers

4 consecutive multiples of 3, 4, 5...

Decimals that differ by 1, such as 1.2, 2.2, 3.2, 4.2

Four numbers going up in 3s, such as 2, 5, 8, 11

Four numbers going up in $\frac{1}{2}$ s, such as 4, $4\frac{1}{2}$, 5, $5\frac{1}{2}$

Make up a few similar questions of your own. Impress your friends by giving them a calculator and 'predicting' what will happen!

NOTES FOR TEACHERS

SOLUTION

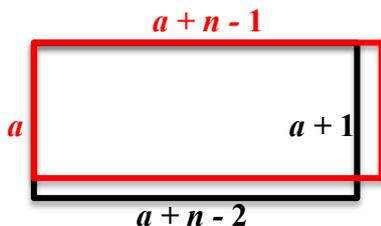
For 4 consecutive numbers the difference of the pair products is always 2.

For 5 consecutive numbers the difference of the pair products is always 3.

For 6 consecutive numbers the difference of the pair products is always 4.

The product of the first and last numbers is shown by the area of the red rectangle.

The product of the second and next to last numbers is shown by the area of the black rectangle.



The difference of the pair products is shown by the difference in the areas of the red and black rectangles.

A strip of area $(a + n - 2) \times 1$ is added on
And a strip of area $a \times 1$ is taken away.

Whatever the first term a the difference of the pair products is always $n - 2$.

Conjecture: For n consecutive numbers the difference of the pair products is always $n - 2$.

Proof If the first number is a then the n th consecutive number is $a + n - 1$.

The product of the first and last numbers is $a(a + n - 1) = a^2 + an - a$

The product of the second and next to last numbers is

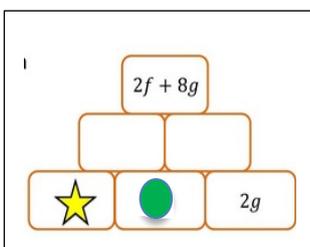
$$\begin{aligned} (a + 1)(a + n - 2) &= a^2 + an - 2a + a + n - 2 \\ &= a^2 + an - a + n - 2. \end{aligned}$$

The difference of these two pair products is $(a^2 + an - a + n - 2) - (a^2 + an - a) = n - 2$.

Diagnostic Assessment

This should take about 5–10 minutes.

- Write the question on the board, say to the class:
“Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D”.
- Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
- Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
- Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers. It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
- If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.



Each block contains the sum of the expressions in the 2 blocks below it.

Tom says the star is f and the circle is $f + 3g$.

Kate says the star is $2f$ and the circle is $3g$.

Who is right?

- Only Tom
- Only Kate
- Both Tom and Kate
- Neither Tom nor Kate

Correct answer: B

- With Tom's choice the top box would become $3f + 8g$
- With Kate's choice the other boxes would contain $2f + 3g$ and $5g$ Giving $2f + 8g$ in the top box as required.
- and D. are wrong.

Why do this activity?

For younger learners this activity can be tackled purely numerically, as an exercise in developing fluency with multiplication tables while looking for pattern and structure.

For learners who need practice in the routine algebraic procedure of expanding brackets this activity is a good introduction to the power of algebra. The related geometrical interpretation helps to deepen understanding of the algebra. The activity gives an excellent context for observing, conjecturing and thinking about proof.

Learning objectives

In doing this activity students will have an opportunity

For Years 7 and 8: to practise investigating a process and making a conjecture from numerical evidence.

For Years 9 to 11:

- to experience generalising a number pattern by writing it as an algebraic expression
- to practise expanding brackets
- to experience making and proving a conjecture using algebra.

Generic competences

In doing this activity students will have an opportunity to:

- **think flexibly**, be creative and innovative and apply knowledge and skills.

Suggestions for teaching

You may like to use [the NRICH poster](#) for this activity.

"Choose four consecutive numbers, multiply the outer pair and the inner pair. What were your two answers?"

Write a selection of students' responses on the board.

"What do you notice?" *The inner pair product is always two more than the product of the outer pair.*

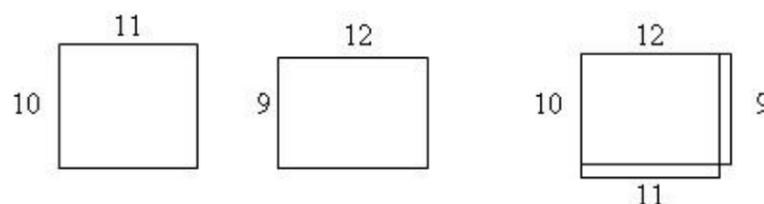
"Will this always happen? Can you explain why?"

Give students some time to discuss with their partner why the answers always differ by two.

Circulate and listen for interesting insights. Bring the class together and share any explanations they have found.

For younger learners you might choose to stop here, or to repeat the investigation for 5 or 6 consecutive numbers.

For older learners you may want to pursue the algebraic proof. If nobody has suggested the



geometrical explanation then you could introduce this diagram for the learners to discuss.

Similarly you could introduce algebra to prove the result.

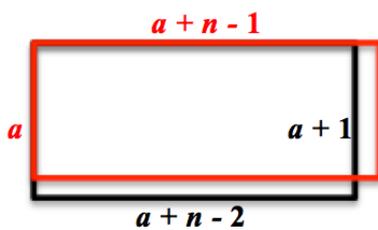
Then go on to generalise the problem.

"We've worked out what happens when you find the product of the inner and outer pair of a set of four consecutive numbers. What questions do you think a mathematician might ask next?"

Write students' suggestions up on the board. If suggestions are not forthcoming, introduce some of the ideas listed in the problem.

"You should be able to work out what will happen in the situations you've suggested using one of the powerful representations we've looked at, without having to try out lots of numerical examples first. Of course, if you want, you can use a numerical example to verify what you've done."

Students could be offered a choice of which situations to work on. Alternatively, you may want everybody to work on a series of related problems (5, 6, 7 ... n consecutive numbers, for example) that will lead to a generalisation.



You could use this diagram in the summing up at the end of the lesson to help learners to explain the proof.

One nice plenary activity is to challenge students to work out quickly what the difference in pair products will be for a randomly chosen sequence of numbers.

You might be interested in this discussion of using this activity with different age groups.

<https://casmusings.wordpress.com/2012/05/13/multiplication-puzzle-for-the-very-young/>

Key questions

Is there a way to represent the pair products that will explain the patterns you noticed?

Follow up

Use area to find x <https://aiminghigh.aimssec.ac.za/years-7-9-use-area-to-find-x/>

Algebrarea

<https://aiminghigh.aimssec.ac.za/years-7-10-algebrarea-product-of-two-brackets-and-area/>

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6. The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA. For resources for teaching A level mathematics see <https://nrich.maths.org/12339>

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6