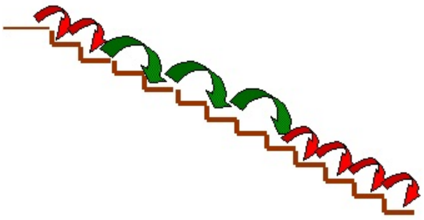
## ONE STEP TWO STEPS

A staircase has 12 stairs. You can go down the stairs one at a time or two at time.



For example: You could go down 1 step (as shown in red), then 1 step, then 2 steps (as shown in green), then 2, 2, 1, 1, 1, 1 steps at a time, as in the picture.

In how many different ways can you go down the 12 stairs, taking one or two steps at a time?

Hint: Do the question for a smaller number of stairs and see if you can find a pattern.
How many ways can you go down 3 stairs?
What about 4 stairs?
What about 5 stairs ...?

## HELP

Learners should work on their own to find as many arrangements as they can with 6 stairs, then compare their answers with a partner to see if either of them has answers the other has not found.

Then each pair should compare answers with another pair. Finally, the class should list all the answers that they have found between them and try to list them in a systematic order.

## NEXT

| **n** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15... |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **$S_n$** | 1 | 1 | 2 | 3 | 5 | 8 | 13 | 21 | 34 | 55 | 89 | 144 | 233 | 377 | 610... |

Start from $S_3 = 2$ and list every 3rd number in the sequence. What do you notice?
Start from $S_4 = 3$ and list every 4th number in the sequence. What do you notice?
Start from $S_5 = 5$ and list every 5th number in the sequence. What do you notice?

## NOTES FOR TEACHERS

## SOLUTION

| | | |
|---|---|---|
| 1 Stair | 1 | Total 1 way |
| 2 Stairs | 1,1 or 2 | Total 2 ways |
| 3 Stairs | 1,1,1 or 1,2 or 2,1 | Total 3 ways |
| 4 Stairs | 1,1,1,1 or 1,1,2 or 1,2,1 or 2,1,1 or 2,2 | Total 5 ways |
| 5 Stairs | 1,1,1,1,1 or 1,1,1,2 or 1,1,2,1 or 1,2,1,1 or 1,2,2 or 2,2,1 or 2,1,2 or 2,1,1,1 | Total 8 ways |
| 6 Stairs | 1,1,1,1,1,1 or 1,1,1,1,2 or 1,1,1,2,1 or 1,1,2,1,1 or 1,2,1,1,1 or 1,1,2,2 or 1,2,2,1 or 2,1,1,2 or 2,1,2,1 or 2,2,1,1 or 2,2,2 or 2,1,1,1,1 or 1,2,1,2 | Total 13 ways |
| etc. | | |

**Each total is the sum of the previous two totals.**

If we use $S_n$ for the number of ways of going down n stairs then $S_{n-1}$ is the number of ways for (n – 1) stairs and $S_{n-2}$ is the number of ways for (n – 2) stairs.

So the rule is $S_n = S_{n-1} + S_{n-2}$

The reason is that at the top you have 2 choices

(1) Go down 1 stair then there (n – 1) stair to go and you have $S_{n-1}$ ways of going down them or
(2) Go down 2 stair then there (n – 2) stairs to go and you have $S_{n-2}$ ways of going down them.

So to find how many ways altogether you need to add all the ways for these two choices so $S_n = S_{n-1} + S_{n-2}$.

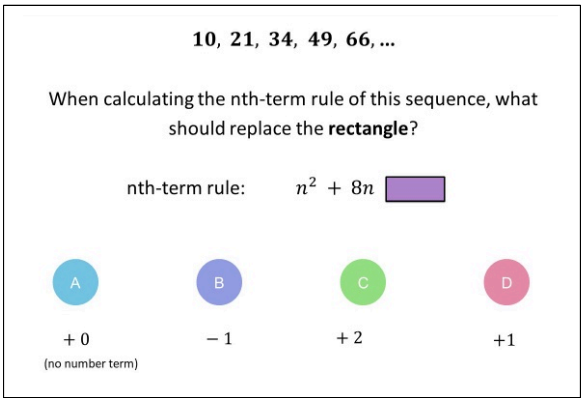
So for more stairs we use the sequence 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, … giving the answer 233.

These are the Fibonacci numbers starting at 1, 2, … from the Fibonacci sequence:
1, 1, 2, 3, 5, 8, 13, …

This links to a lot of mathematics about The Golden Ratio with applications and examples in nature, in art and in geometry.

## DIAGNOSTIC ASSESSMENT FOR YEARS 9 TO 13

This should take about 5–10 minutes.
Write the question on the board, say to the class:
**"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".**




1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reasons for their answers. Putting thoughts into words may help them to gain better understanding and improve their communication skills.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the class to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

The correct answer is:
Possible misconceptions:

## Why do this activity?

This activity will help learners to develop problem solving skills, in particular to start with simple cases for 1 step, then 2, then 3, then 4 etc. and to look for a pattern in the sequence of answers. Working on this problem also gives learners an experience of the advantage of working systematically.

Learners will meet arithmetic and geometric sequences where the $n^{th}$ term can be written as a formula involving n. Unlike these sequences the terms of the sequence for this activity 1, 2, 3, 5, 8, … do not change in equal steps or with a common ratio. To meet a sequence that is defined quite differently, and one that is found widely in different examples in nature, in art and in geometry, will broaden their knowledge and appreciation of mathematics.

The problem also involves numbers of different arrangements or permutations, a topic which they will meet later in school, but here it is met it in a sufficiently simple way to make it accessible at this level.

## Learning objectives

In doing this activity students will have an opportunity to:

- investigate numeric and geometric patterns looking for relationships between numbers;
- extend a sequence following a pattern;
- develop problem solving skills;
- make and prove a conjecture about the formula for a Fibonacci sequence.

## Generic competences

In doing this activity students will have an opportunity to:

- develop problem solving skills;
- make a plan and work systematically to find all possible solutions to a given problem.

## Suggestions for teaching

Tell the class what the problem is. Show them the picture and talk about how it shows going down in steps of 1, 1, 2, 2, 2, 1, 1, 1, 1 steps at a time making 12 steps in all. (Note: the language used, a step is what you do with your feet, a stair is part of a staircase. We talk about a step ladder and rungs on the ladder.)

Ask learners to suggest other examples for 12 steps starting with 1, 2, … and 2, 1,… and agree that there must be a large number of different possibilities.

Then suggest that the class should start with simple cases, with 1 step, then 2 steps, then 3 steps etc and look for a pattern.

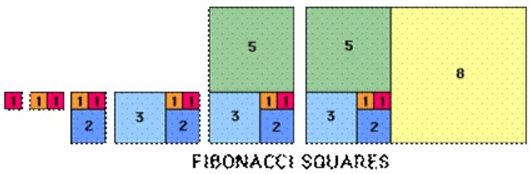
Give them a few minutes to work out the number of steps for n = 1, 2 and 3 then agree the answers as a class so that everyone is helped to understand what is involved.

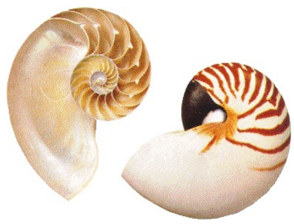
Then ask the class to work out the answers for 4, 5 and 6 stairs.

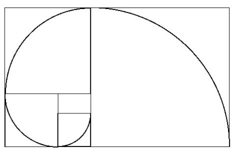
Finally ask the learners to explain their answers for 4, 5 and 6.

Then write down the sequence 1, 2, 3, 5, 8, 13 … on the board and ask the learners to look for a pattern.

When someone spots the pattern they must explain it to the class. Then the class should continue the sequence 1,2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233 … to find the answer for 12 steps :- 233.





Here are some examples showing the Fibonacci sequence in nature and art. If the class has access to the internet they might do some searches for themselves, to find out about the Golden Ratio.

See http://nrich.maths.org/4836

## Key questions

- Are you sure that you have listed all the possibilities? How do you know?
- Have you listed the possibilities in a special order?
- How about listing the steps taken starting with 1 step, then the steps taken starting with 2 steps.
- Look at any three successive terms, do you notice anything?

- Is there a pattern in the sequence 1, 2, 3, 5, …?
- What do you think the next term might be? Can you make a list of the possibilities and check it?

## Follow up

Elephant Dreaming https://aiminghigh.aimssec.ac.za/years-8-12-elephant-dreaming/

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za

Subscribe to the **MATHS TOYS YouTube Channel** https://www.youtube.com/c/mathstoys

Download the whole AIMSSEC collection of resources to use offline with the **AIMSSEC App** see https://aimssec.app or find it on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

| | Lower Primary Approx. Age 5 to 8 | Upper Primary Age 8 to 11 | Lower Secondary Age 11 to 15 | Upper Secondary Age 15+ |
|---|---|---|---|---|
| South Africa | Grades R and 1 to 3 | Grades 4 to 6 | Grades 7 to 9 | Grades 10 to 12 |
| East Africa | Nursery and Primary 1 to 3 | Primary 4 to 6 | Secondary 1 to 3 | Secondary 4 to 6 |
| USA | Kindergarten and G1 to 3 | Grades 4 to 6 | Grades 7 to 9 | Grades 10 to 12 |
| UK | Reception and Years 1 to 3 | Years 4 to 6 | Years 7 to 9 | Years 10 to 13 |