OLYMPIC RINGS

The Olympic emblem consists of five overlapping rings containing nine regions.
Place the numbers 1, 2, 3, ... 9 in the nine regions so that the total in each ring is the same.
Can you complete the solutions given in the 4 diagrams?

SOLUTION

A. The sum $1 + 2 + ... + 9 = 45$ and 3, 1, 4 and 2 are counted twice so the total for all 5 rings is $45 + (1 + 2 + 3 + 4) = 55$ so the total in each ring is 11.

B. The numbers to go on the overlapping regions are 2, 4, 6 and 8 which add up to 20. These numbers are counted twice so the total for all 5 rings is 65 and the total in each ring is 13.

C. Putting in $a$ and $b$ in the overlapping regions we have $5 + a = 8 + b$ so $a - b = 3$.
Then $5(5 + a) = 45 + a + b + 10$ so $4a - b = 30$
Solving these two simultaneous equations we get $a = 9$ and $b = 6$ so the total in each ring is 14.

D. Putting in $c$ and $d$ in the overlapping regions we have $4 + c = 7 + d$ so $c - d = 3$.
In this example $5(4 + c) = 45 + c + d + 5$ so $4c - d = 30$
Solving these two simultaneous equations we get $c = 9$ and $d = 6$ so the total in each ring is 13.
NOTES FOR TEACHERS

Why do this activity?
In this activity learners get practice in analysing the mathematical features of the information given :- equal totals, arrangements of the numbers from 1 to 9, these numbers add up to a total of 45, certain numbers are used twice and others once. Learners could use trial and error to get the solutions but should also think creatively to find more systematic methods.

Intended Learning Outcomes Grades 9 to 12)
To gain confidence in reading the question and thinking about how to use the information given to find solutions. To share ideas and improve communication and presentation skills. To understand how simultaneous equations can be used to solve this problem.

Possible Approach
You might like to plan your lesson to give the 4 different examples A, B, C and D to different groups of learners and then have each group make a poster of their solution to present to the class. You could give C and D to the more able learners as the best way to solve these two examples is using simultaneous equations and the other two examples are more straightforward.

With younger learners you might just give the class examples A and B and only use C and D as extension activities for high flyers.

Some groups will use trial and error. Let them do so for a while. If they succeed you can congratulate them and then use the Key Questions to guide them in looking for another method. If they struggle to find solutions by trial and error you can use the Key Questions to guide them in looking for another method.

Key questions
Which numbers are missing?
Which numbers are used once and which are used twice when the numbers in the 5 circles are added up?
What is the sum of the whole numbers from 1 to 9?
What can you say about the total of the numbers that are in the intersections of the circles?
If the total in each circle is X what can you say about 5X?
Can you write down equations using two letters for the two unknown numbers in the overlapping regions?

Possible extension
Some learners can do one or two of the examples A and B and others can be given C and D as extensions. It is possible to find solutions and to show that these are the only 4 solutions without being given any of the positions of the numbers. See http://nrich.maths.org/460/solution

Possible support
Learners could work on some arithmagon puzzles. See Chapter 9 of the book AIMSSEC Mathematical Thinking in the Lower Secondary Classroom.

Put numbers into the circles so that the number in each square is the sum of the numbers in the two circles next to it. Hint:
In (a) the numbers are x, 8 – x and 9 - x