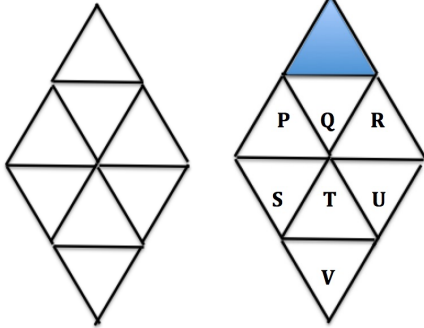


## MATCHES



Find some sticks, they could be twigs, or matchsticks or cocktail sticks, or pasta broken into short lengths, or rolled paper sticks, but they must all be the same length. Arrange 16 sticks into this pattern.

What symmetries do you see in the pattern?

The triangles are all equilateral. What transformations map the blue triangle to the other triangles P, Q, R, ... ?

How many triangles can you see in this pattern?

**Here is a puzzle for you with more than one solution.**

In how many ways can you produce a pattern with exactly 4 triangles by removing exactly 4 matches and leaving the others in place?

Some of these patterns are reflections or rotations of each other.

How many essentially different patterns are there counting reflections and rotations of a pattern as the same?

## HELP

Make the pattern with sticks, any sticks that you can find. This will help you with visualisation so that you can see what happens when you remove sticks from the pattern.

Make a paper triangle that you can move around to see what happens when you rotate the triangle or flip it over (reflect it).

## NEXT

The activity Quarters <https://aiminghigh.aimssec.ac.za/years-4-10-quarters/>

## NOTES FOR TEACHERS

### SOLUTION

The pattern has 2 lines of symmetry one vertical and one horizontal.

It has 2-fold rotational symmetry about the centre point.

The blue triangle maps to P by a translation and also by a clockwise rotation of  $120^\circ$  about its bottom left vertex.

It maps to Q by a reflection in its base or by a rotation of  $180^\circ$  about the midpoint of the base.

It maps to P by a translation and also by a clockwise rotation of  $120^\circ$  about its bottom left vertex.

It maps to R by a translation and also by an anti-clockwise rotation of  $120^\circ$  about its bottom right vertex.

It maps to S by rotation of  $180^\circ$  about a point in the centre of P.

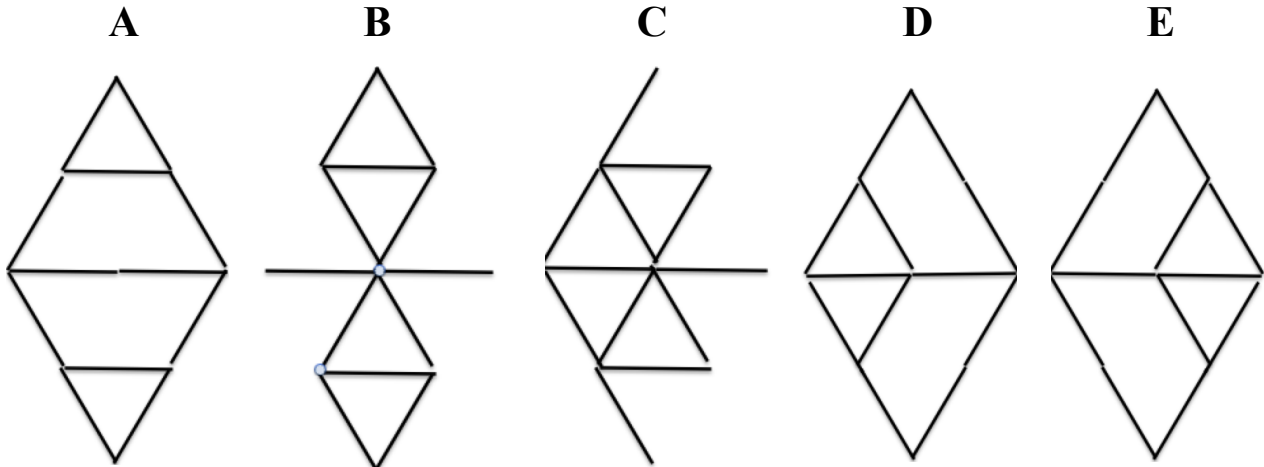
It maps to T by a translation.

It maps to U by rotation of  $180^\circ$  about a point in the centre of R.

It maps to V by reflection in the middle line.

There are 10 triangles in the original picture, 2 large triangles and 8 smaller congruent triangles.

See below for 4 solutions that give 4 triangles when 4 matches are removed. D and E are reflections in a vertical line so they are not distinct solutions. Can you see any other solutions?



### Why do this activity?

This activity gives learners the experience of exploring the symmetries in a pattern and describing what they see in it. Solving the puzzle and considering how some possible solutions are reflections of other solutions is another exercise on transformations.

### Learning objectives

In doing this activity students will have an opportunity to:

- develop understanding of symmetry and the ability to describe it;
- use transformations to make composite shapes, including shapes with line symmetry, by moving a 2-D shape in one or more of the following ways:- by rotation, by translation, by reflection.

## Generic competences

In doing this activity students will have an opportunity to:

- **think flexibly**, be creative and innovative and apply knowledge and skills;
- **develop visualization** and skill to interpret or create images to represent concepts and situations.

### DIAGNOSTIC ASSESSMENT

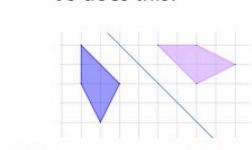
This should take about 5–10 minutes. It can be used before or after the lesson.

Show the question to the learners and say:

**“Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D”.**

Jo and Paul are arguing about how to reflect the object in the line of reflection. Who reflects it correctly?

Jo does this:



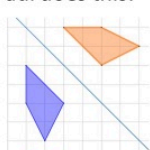
A

Only  
Jo

B

Only  
Paul

Paul does this:

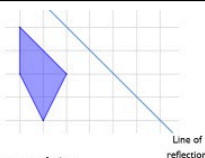


C

Both Jo  
and Paul

D

Neither is  
correct



1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reason for their answer because it helps them to clarify their own thinking and to develop communication skills.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.

4. Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.

**The correct answer is: B**

Possible misconceptions: A. Jo has done a reflection followed by a translation (a glide reflection).

<https://diagnosticquestions.com>

## Suggestion for Teaching

The concrete experience of physically making the pattern with sticks helps learners to visualise it. Similarly experimenting with a paper triangle helps learners to see what happens when they rotate the triangle or flip it over (reflect it).

You may use this activity as an example when you are teaching about transformations. Alternatively, if learners have prior knowledge of transformations, you may use it to deepen their understanding and gain experience of describing the actions of transformations.

If possible it is best to give learners sticks of some sort. If you use matches use dead ones. The learners should be given the question on paper, or read it from the board, and then working in pairs or groups should make the pattern for themselves and make a careful record of all their discoveries.

Give the learners a triangle cut from paper, or ask them to cut one out themselves, so that they can move it around to explore the transformations from one inner triangle to another. They can remove sticks from the framework to find the different solutions. They could record their solutions by drawing them and make notes about the transformations.

When the class have had sufficient time to investigate the symmetries of the pattern and to find solutions to the puzzle, then conduct a class discussion of the answers using a diagram on the board and a paper triangle of the right size.

If they have not found all 4 solutions **don't give the answers**. Tell them that there are 4 different solutions and challenge them to go on trying to find them. **Perseverance is an important life skill**.

At the end of the lesson, based on what they have experienced, help the learners to make a summary of what they know about symmetries and transformations.

### **Key questions**

- Can you see big triangles as well as small triangles?
- If you rotate the triangle which point does it turn around, which point is the centre of rotation.
- If you rotate the triangle from there to there what angle does it turn through? Is that clockwise or anti-clockwise?
- If you reflect the triangle where is the mirror line? Show me? Can you describe it?
- If you translate the triangle what direction does it go? Show me? Can you describe it?
- How far is it translated if one matchstick is 1 unit in length?

### **Follow up**

Quarters <https://aiminghigh.aimssec.ac.za/years-4-10-quarters/>