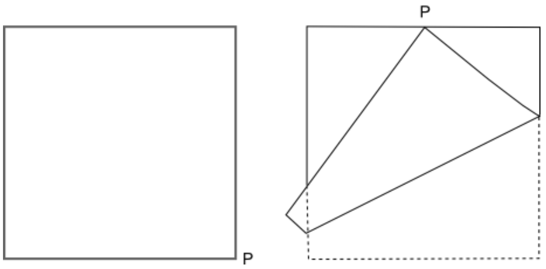
AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

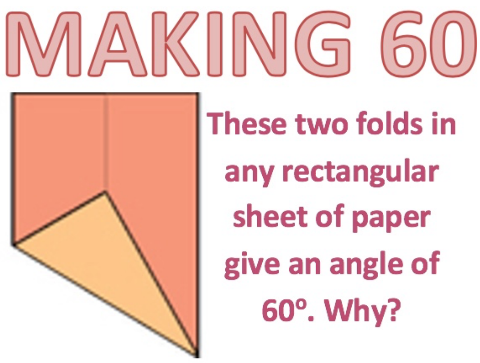
FOLD A SQUARE



A square of paper 8 cm by 8 cm is folded so that the corner P coincides with the midpoint of an opposite edge as shown in the diagram.

Investigate the three triangles (where there is a single thickness of the paper) that are formed by folding in this way.

HELP



If you find it difficult to solve to find the lengths and angles in the problem above then try this question first. It is a similar context involving a similar fold but with more accessible results. It can also lead to some practical mathematics, making an equilateral triangle just by paper folding.

https://aiminghigh.aimssec.ac.za/years-9-10-making-sixty/

NEXT

If the square of paper is folded so that the corner P does not coincide with the midpoint of an opposite edge, where would you place the fold for a

5, 12, 13

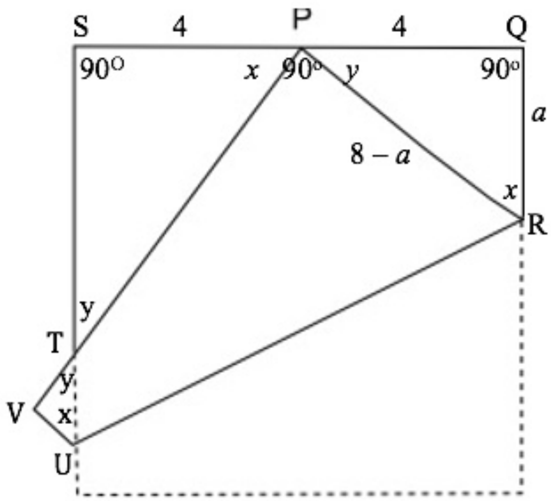
or an 8, 15, 17

or a 7, 24, 25 triangle?

Are any of these findings extendable to other quadrilaterals?

NOTES FOR TEACHERS

SOLUTION



Triangles PQR, SPT and TUV are all right angled triangles because one of their angles was at the corner of the square.

If PQ = 4 (because P is the midpoint of SQ)

As QR = a then PR = 8 – a.

By Pythagoras Theorem for triangle PQR

$16 + a^2 = (8 - a)^2$

$16 + a^2 = 64 - 16a + a^2$

$16a = 48$

$a = 3$.

So PQR is a 3-4-5 triangle with angles $x = \cos^{-1}0.6 = 53^o$ and $y = \sin^{-1}0.6 = 37^o$ to the nearest degree.

From the angles on the straight line at P we see that ∠SPT = ∠QRP = x^o.

So ΔSPT is right angled it is similar to ΔQRP.

Also ∠PTS = ∠UTV (vertically opposite angles)

Δs PQR, TSP, TVU have angles *x, y* and 90° and are similar 3-4-5 triangles.

Hence ST/4 = 4/3 and ST = 5⅓ and
PT/5 = 4/3 so PT = 6⅔ and TV = 8 - 6⅔ = 1⅓.
Then VU= 1 and TU = 5/3.

Triangle	Lengths of sides	Ratio of lengths in 3 triangles	Area	Ratio of areas in 3 triangles
PQR	3 : 4 : 5	3	6	9
TSP	4 : 16/3 : 20/3	4	32/3	16
TVU	1 : 4/3 : 5/3	1	2/3	1

This problem can also be solved using trigonometry:

$ST = 4 \tan x = 4 \times 4/3 = 16/3$

$PT \sin y = 4$ so $PT = 4/(3/5) = 20/3$

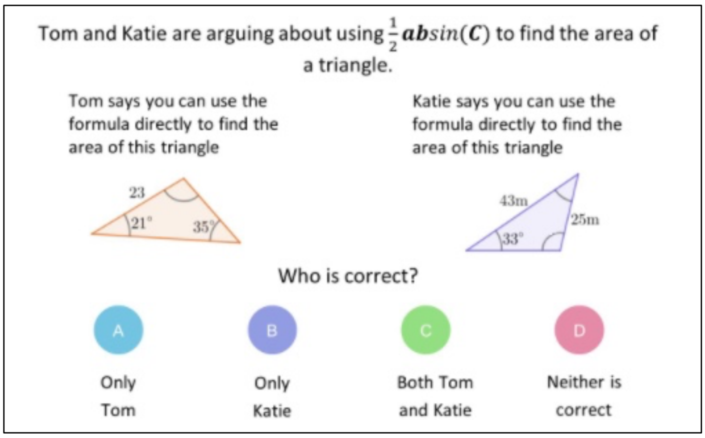
$VU = TV \tan y = (4/3)(3/4) = 1$

$TU \sin y = VU$ so $1/\sin y = 5/3$

DIAGNOSTIC ASSESSMENT This should take about 5–10 minutes.

Write the question on the board, say to the class:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".



1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reasons for their answers. Putting thoughts into words may help them to gain better understanding and improve their communication skills.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the class to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.

The correct answer is: D

A: Tom is wrong because the formula needs ***two edge lengths*** and this diagram only shows one. If a is the length of the base, then b sin C is the height of the triangle.

B: A: Katie is wrong because the formula needs the lengths of two edges and the angle ***between*** those edges so that if a is the length of the base, then b sin C is the height of the triangle.

https://diagnosticquestions.com/

Why do this activity?

This activity provides a simple context in which to generate lots of questions, make conjectures and form convincing arguments. It is perhaps surprising that all the triangles turn out to be 3-4-5 triangles. The activity brings together important geometrical concepts: recognising similar triangles, calculating lengths and Pythagoras Theorem.

Learning objectives

In doing this activity students will have an opportunity to:

- review Pythagoras Theorem and properties of right angled triangles;
- review areas of triangles;
- review ratios of lengths and area under enlargement;
- review similar triangles
- review basic trigonometry of right angled triangles.

Generic competences

In doing this activity students will have an opportunity to:

- develop flexible thinking;
- develop the skill of visualization, literally imagining a problem and seeing it as an mental image.

Suggestions for teaching

At the start encourage learners to list and share what they notice using a square of paper. Using large squares of paper on a display board can encourage discussion of key features, ideas and conjectures which learners might explore.

Identify questions about the triangles that learners will work on.

It is likely that learners will arrive at results in different ways. These journeys and findings form opportunities to share and discuss good and elegant solutions, and to develop communication skills by explaining their ideas to the class. This activity also helps learners to develop different ways of "seeing" the problem. The best problem solvers are those who think visually, in pictures.

Key questions

- What do you think might be true?
- What do you know?
- What do you need to know?
- What mathematical ideas and techniques might be of use in order to answer that question?

Follow up

Making Sixty https://aiminghigh.aimssec.ac.za/years-9-10-making-sixty/

Investigating Circle Theorems
https://aiminghigh.aimssec.ac.za/years-10-11-investigating-circle-theorems/

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za
Subscribe to the **MATHS TOYS YouTube Channel** https://www.youtube.com/c/mathstoys
Download the whole AIMSSEC collection of resources to use offline with the **AIMSSEC App** see https://aimssec.app or find it on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13