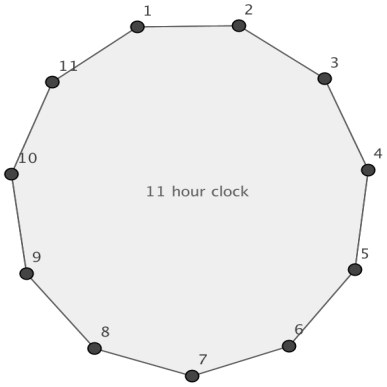
AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE
TEACHER NETWORK

ELEVENSES



9	46	79	13
64	90	2	97
25	31	20	22
4	52	55	7

In this grid, how many pairs of numbers can you find that add up to a multiple of 11?

Do you notice anything interesting?
Can you find all the pairs?
Could you convince someone that you haven't missed any?

Can you use the same ideas to find all the pairs of numbers that add up to multiples of 13? This time there are not as many pairs.

SOLUTION

By writing all the numbers as multiples of 11 plus a remainder we can pair them up to make multiples of 11.

Remainder on division by 11		Pairs of numbers that add up to a multiple of 11. **There are 28 pairs in total.**
0	22, 55	These numbers add up to a multiple of 11
2	2, 13, 46, 79, 90	These numbers with any of the numbers that have a remainder 9. 25 pairs
3	25	25 + 52 = 77
4	4	4 + 7 = 11
7	7	7 + 4 = 11
8	52	52 + 25 = 77
9	9, 20, 31, 64, 97	These numbers with any of the numbers that have a remainder 2

Remainder on division by 13		Pairs of numbers that add up to a multiple of 13 **There are 9 pairs in total.**
0	13, 52	These numbers add up to a multiple of 13
1	79	This number with any of the numbers that have a remainder 12
2	2	No combinations involving 2
3	55	No combinations involving 55
4	4	This number with any of the numbers that have a remainder 9
5	31	No combinations involving 31
6	97	This number with any of the numbers that have a remainder 7
7	7, 20, 46	These numbers with 97 which has a remainder 6
9	9, 22	These numbers with 4
12	25, 64, 90	These numbers with 79 which has a remainder 1

NOTES FOR TEACHERS

Why do this activity?

On first inspection this appears to be an opportunity to practice mental calculation strategies, but it soon becomes apparent that this context offers an opportunity to think about the structure of numbers, about multiples and clock arithmetic, and the about division and remainders.

Possible approach

Write the numbers and draw the 11 hour clock on the board or give out copies of the problem. Allow a little time for the learners to find a couple of pairs that add to a multiple of 11. Collect suggestions and display them on the board. Set the challenge - how many pairs can they find? Can anyone find them all?

When they are well into the problem, stop them and ask "What have you noticed about the pairings?" Collect ideas and note them on the board. If no one has suggested it, draw attention to the pairings involving 9, 20 and 31. "What is special about these numbers? What is special about their 'partners'?"

Suggest that they return to the problem and use this insight to find out how many pairings are possible. Can this be done without listing them?

When appropriate, bring the class together and draw out ideas that lead to an efficient strategy. Discuss the way clock arithmetic occurs wherever there are cycles such as days of the week, the 24-hour clock, or minutes and seconds.

Key questions

Can you use the 11-hour clock to help find the pairs?
What can you combine with 9?
What can you combine with 20?
What can you combine with 31?
What is special about these numbers?
What is special about their partners?
What is special about the numbers in each pairing?
Are there some numbers that can only be used once? Why?
Are there some numbers that can be used many times? Why?

Possible extension

Both grids contain less than 30 possible pairings. Can you produce a grid of numbers that has more than 30?
What is the maximum number of possible pairings in a 4x4 grid?
Legs Eleven https://aiminghigh.aimssec.ac.za/grades-10-to-12-legs-eleven/ may provide an interesting follow-up challenge.

Possible support

The grid below could be used to ask learners to find pairs that add to a multiple of 10.
The key questions are useful prompts to focus students on the structure of the numbers rather than multiple calculations. This could be useful preparation before going on to the main problem.

8	42	72	12
58	82	2	88
23	28	18	20
4	47	50	6