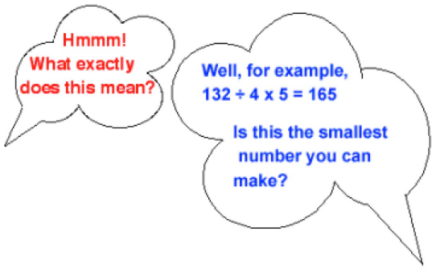
DOWNSIZE



Using the numbers 1, 2, 3, 4 and 5 once and only once and the symbols × and ÷ once and only once, what is the smallest whole number you can make?

Now suppose you have to use the symbol + once and the symbol ÷ twice, what is the smallest fraction that you can find?

For example $\frac{1}{4} + \frac{2}{35} = \frac{43}{140}$.

Can you find a smaller fraction?

HELP

This activity is about developing and using number sense and understanding numbers rather than doing calculations. If it helps you could use a calculator.

NEXT

Make the largest whole number following the same rules.

How many whole numbers can you make in this way?

NOTES FOR TEACHERS

SOLUTION

There are several ways to get the answer 1, the smallest possible result.

(1) $4 \times 13/52 = 4 \times ¼ = 1$ (or $13/52 \times 4$ which is the same because multiplication is commutative.)

(2) $13 \times 4/52 = 13 \times 1/13 = 1$ (or $4 \times 13/52$ which is the same because multiplication is commutative.)

Note: $52/(13 \times 4) = 1$ requires brackets, because without brackets the answer is 16.

Can you find a smaller fraction than $\frac{1}{5} + \frac{2}{43} = \frac{53}{215}$? If not, how do you know that this is the smallest?

To compare the fractions learners could use long division or a calculator.
Method 1 $43/140 = 0.307$ to 3 decimal places and $53/215 = 0.247$ to 3 decimal places so $53/215 < 43/140$.

Method 2
$53 \times 140 = 7420$ and $43 \times 215 = 9245$ so
$53 \times 140 < 43 \times 215$ and it follows that
$\frac{53}{215} < \frac{43}{140}$

Why do this activity?

This activity could be used to start a lesson, or as an extension activity for those who finish other tasks early so the teacher can plan for learners of different abilities in the class. It will promote thinking about numbers and it offers opportunities to practise multiplication, division and fractions.

Intended Learning Objectives (Years 4 - 6)

The development of number sense that includes:

- the meaning of different kinds of numbers;
- relationship between different kinds of numbers;
- the relative size of different numbers;
- representation of numbers in various ways;
- the effect of operating with numbers;
- the ability to estimate and check solutions.

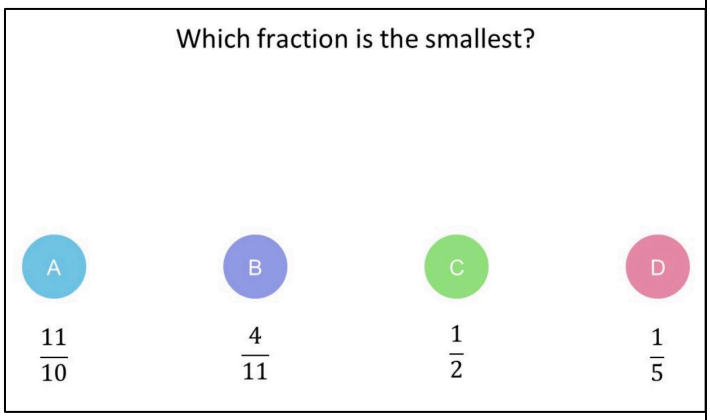
Generic Competences

In doing this activity learners have the opportunity to develop the skill of planning and working systematically to find the best possible solution in one case and then to find all possible solutions.

Diagnostic Assessment This should take about 5–10 minutes.

Write the question on the board, say to the class:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".



1. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reason for their answer as it helps others in the class to reflect on the best answer, and helps them to develop communication skills; also, putting thoughts into words, helps one to think clearly.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.**
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task. **D. is the correct answer.**

Common Misconceptions

A. Learners who give this answer do not understand that such fractions are greater than 1.
B. Learners who choose this may know that B it is less than A and C but not be able to compare it to D.
C. Learners who give this answer have no understanding about what sort of fractions are less then ½.
https://diagnosticquestions.com

Suggestions for teaching

If you use the whole number activity as a lesson starter, write the question on the board, perhaps before the learners come into the classroom. Tell the learners to work on it on their own without talking. Then after a few minutes tell them to compare their answers in pairs. Then the whole class should share answers, writing them on the board in decreasing size. It is important for the learners to explain how each answer is calculated.

Use the diagnostic assessment question before the class starts to find the smallest fraction.

You could follow a similar procedure with the smallest fraction challenge as with the smallest whole number challenge. Remind the class that they can only use each of the five digits ONCE only. Ask the learners to work in pairs or small groups to make the smallest fraction that they can.

Comparing the fractions to find which is smaller and which is bigger is a good exercise in itself. The learners might use long division or a calculator to find which is the smallest. Learners might find the cross multiplying method or you could suggest it.

Key questions

- What results can you find obeying these rules?
- Why don't you put all your answers in order?
- What things do you notice about your different results?
- Why do you not get any divisions by 5?
- Does it help to multiply by 1? If not, why not?
- What can you do to the denominator to make the fraction smaller?
- What can you do to the numerator to make the fraction smaller?

Follow up

Tangram Fractions https://aiminghigh.aimssec.ac.za/years-5-9-tangram-fractions/
Fractions in a Square
https://aiminghigh.aimssec.ac.za/years-4-7-fractions-in-a-square/
Fraction Wall https://aiminghigh.aimssec.ac.za/years-4-6-fraction-wall/
Chocolate https://aiminghigh.aimssec.ac.za/years-4-8-chocolate/
Fractions by Thirds https://aiminghigh.aimssec.ac.za/years-4-7-fractions-by-thirds/
Fractions by halves https://aiminghigh.aimssec.ac.za/years-5-6-fractions-by-halves/

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za
Subscribe to the **MATHS TOYS YouTube Channel**
https://www.youtube.com/c/mathstoys
Download the whole AIMSSEC collection of resources to use offline with the **AIMSSEC App** see https://aimssec.app or find it on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13