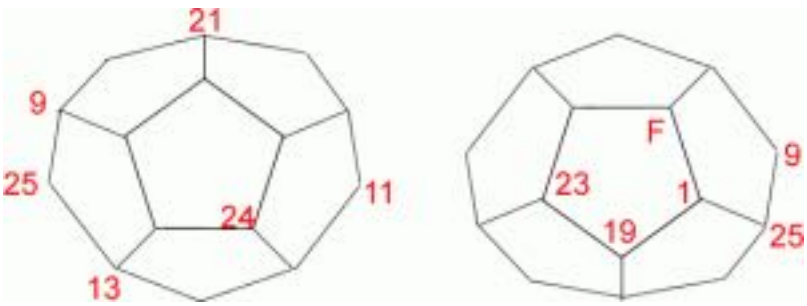
DODECAMAGIC



Here you see the front and back views of a dodecahedron, a solid with pentagonal faces.

Using twenty of the numbers from 1 to 25, each vertex can be numbered so that the numbers around each pentagonal face add up to 65.

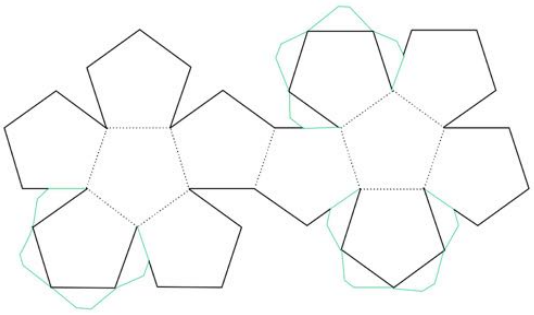
The number F is the number of faces of the solid.

You might like to draw a dodecahedron net and write the numbers at the vertices, or make the solid and write the numbers on it. Can you find all the missing numbers?

Help

To solve this puzzle you need to imagine what objects look like from different angles. Here are two views of a mug looking down from above and looking straight on. The design you can see of the AIMSSEC logo is printed on the reverse side of the mug that is hidden from view. Make a sketch of the view from the back. Make another sketch of the mug from the right with the handle of the mug facing you.

Now try to imagine what the dodecahedron looks like from different angles.

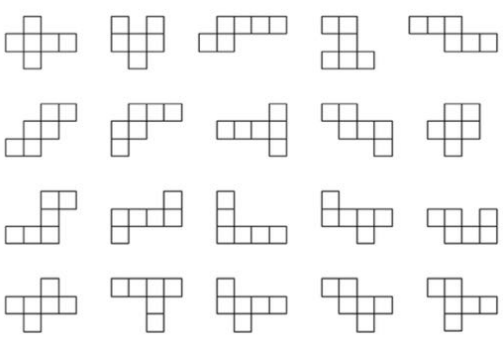


Here is the net of the dodecahedron. Mark the numbers given in the puzzle where you think they should be on the net. Then try to work out where to place the other numbers.

You might make a model of the dodecahedron that you can turn around in your hands to help to solve the puzzle.

Extension

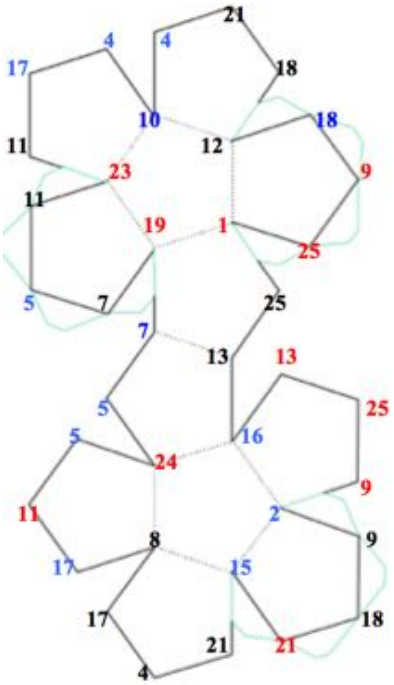
Which of these diagrams is a net that will make a cube and which is not? Choose one, make a cube and then make up your own number puzzle using a cube.



You might try some other model-making activities which will help you to consolidate 2D representation of 3D objects.

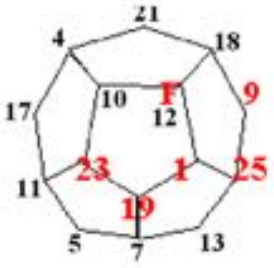
See Cube Nets https://aiminghigh.aimssec.ac.za/years-6-10-cube-nets/
Cut Nets https://aiminghigh.aimssec.ac.za/years-6-10-cut-nets/

SOLUTION



The solution is given on a net of the dodecahedron with the numbers given originally in red, the numbers found from the geometry of the dodecahedron in black and the numbers calculated knowing that the 5 numbers around each face add up to 65 in blue.

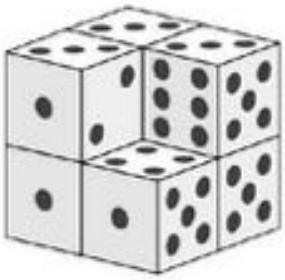
The solution is also given on a copy of the original image.



Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

A large cube is made by stacking eight dice. The diagram shows the result, except that one of the dice is missing. Each die has faces with 1, 2, 3, 4, 5 and 6 pips and the total number of pips on each pair of opposite faces is 7. When two dice are placed face to face, the matching faces must have the same number of pips. What could the missing die look like?



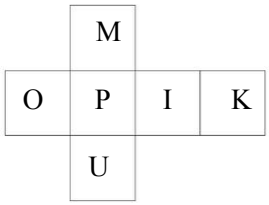
A  B  C  D 

The correct answer is **C**. It must be turned round to put 4 on the bottom, 3 on top, 1 in front, 6 at the back, 2 on the left and 5 on the right.
With A, if 4 is at the bottom and 6 at the back then 5 will be on the left which does not match the 2.
With B, if 2 is on the left and 4 on the bottom, then 1 will be at the back which does not match the 6.
With C, if 6 is at the back and 4 on the bottom, then 5 will be on the left which does not match the 2.

If you cannot easily show the diagrams for the first quiz you may prefer to use the quiz below as it is easy to copy the 3D views on the board.

The diagrams on the right show three different views of the same cube. Which letter is on the face opposite U?

A I B P C M D O



The correct answer is C as is shown from the diagram of the net. The letters O, P, I and K are seen with M in the 3D views. The only letter not seen with M is the letter U. So U must be on the opposite face to M.

https://diagnosticquestions.com

NOTES FOR TEACHERS

Why do this activity?

Visualising is a very important mathematical skill. Representing 3D shapes in a 2D form is a sophisticated form of visualising. This activity offers opportunities to visualise, and to use deductive reasoning with small numbers.

Intended Learning Objectives (Grade 9 and 10)

To develop visualisation of 3D shapes and revise properties and definitions of the 5 Platonic solids in terms of the shape and number of faces, the number of vertices and the number of edges

Generic competences

In doing this activity students will have an opportunity to:

- **think mathematically**, reason logically and give explanations and proofs;
- **think flexibly**, be creative and innovative - to apply knowledge and skills;
- **visualize** and develop the skill of interpreting and creating visual images to represent concepts and situations;
- interpret and **solve problems** in a variety of situations.

Suggestions for teaching

Start the lesson with one of the diagnostic quizzes as a warm up for 3D visualisation.

If you have a set of models of the 5 Platonic solids then use them to review the properties of these solids. Perhaps you could hand them round the class for the learners to handle and then pass on.

If your learners are used to visualising, you may wish to go straight into the problem and see how far they can get. Display the pictures and ensure that everyone understands the problem, then after a little time ask the whole class to share any useful statements or observations that they can make. At this stage you may want to suggest that they could choose to use a 3D model if that would help, but you might also want to challenge them to do it 'in their heads'.

If your learners are not used to visualising, you may want to begin by organising them into pairs to make a 3D model from a net (see below on page 5).

The main challenge is to match the information from one diagram onto the other. Once the learners realise how the two diagrams are connected, the arithmetic is relatively trivial. Labelling the vertices and then opening up the net of the solid can also help them to make the connections.

Key questions

- There are 9s on both diagrams. Does that help? How?
- What about the 25?
- What number does the F represent? How does the name of the shape help you to find F?

Follow-up ideas

The first diagnostic question makes a good activity in itself.

Activities to develop visualisation skills: https://aiminghigh.aimssec.ac.za/years-4-to-9-how-do-you-see-it/ and Three Views https://aiminghigh.aimssec.ac.za/years-4-8-three-views/

Model making activities: https://aiminghigh.aimssec.ac.za/years-7-10-tets-and-octs-puzzles/ https://aiminghigh.aimssec.ac.za/years-7-10-icosahedron-puzzle/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **beyond** the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	**Upper Primary Age 9 to 11**	**Lower Secondary Age 11 to 14**	**Upper Secondary Age 15+**
South Africa	**Grades R and 1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
USA	**Kindergarten and G1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
UK	**Reception and Years 1 to 3**	**Years 4 to 6**	**Years 7 to 9**	**Years 10 to 13**
East Africa	**Nursery and Primary 1 to 3**	**Primary 4 to 6**	**Secondary 1 to 3**	**Secondary 4 to 6**