AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

DIGITS

What is the sum of all the digits in all the integers from one to one million?

Don't do a lot of tedious adding up – think mathematically!

Note: The sum of all the digits in all the integers from one to ten is 46
(not 55 because 10 contributes only 1 to this sum

Hint: Let a million stand alone.

Now introduce zero.

Give every number except one million a **special partner**.

Help

"Think of all the numbers from 0 to 499 999.
Take partners from the numbers from 500 000 to 999 999".

Extension

Try the activity Sandwich Puzzle.
Suppose you have two 1's, two 2's and two 3's.

Arrange these six digits in a list so that:
between the two 1's there is one digit giving 1?1,
between the two 2's there are two digits giving 2??2,
between the two 3's there are three digits giving 3???3.

Can you do the same if you only have 1's and 2's? Explain your answer.
Can you do the same if you include two four's, and between the two 4's there are four digits?
What about two 1s, 2s, 3s, 4s and 5s?

https://aiminghigh.aimssec.ac.za/years-6-to-12-sandwiches/

NOTES FOR TEACHERS

SOLUTION

Look for special partners. What was the reason for bringing in zero? Yes to make up an even number of numbers so each number can have a partner, but what else? Which is the partner for zero? This might be the key that unlocks the whole problem.

It seems natural to start with 0 and 999999 so the sum of their digits is 54.

So what about 1? Well there should be a pattern! What about 1 and 999998? the sum of their digits is 54. That could be useful!

Following this pattern match 2 with 999997, then 3 with 999996 and so on. The sum is 54 for each pair.

How many pairs? Half a million, that is 500 000 pairs.

So the answer is 500 000 × 54 + 1. We have to add 1 as the contribution from 1 000 000.

This gives the total 27 000 001 (twenty seven million and one).

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

$$\begin{array}{r} f\ l\ y \\ +\ f\ l\ y \\ +\ f\ l\ y \\ \hline a\ w\ a\ y \\ \hline \end{array}$$

In this addition sum, each letter represents a different non-zero digit. People found these solutions:

1. $a = 1,\ f = 6,\ l = 0,\ y = 5,\ w = 8$
2. $a = 2,\ f = 8,\ l = 7,\ y = 5,\ w = 6$
3. $a = 2,\ f = 9,\ l = 7,\ y = 5,\ w = 9$
4. $a = 2,\ f = 7,\ l = 7,\ y = 5,\ w = 3$

Which of the following are correct solutions?

A. 1 and 2 **B.** 2 and 3 **C.** 3 and 4 **D.** All of these

The correct answer is B.

$$\begin{array}{r} 875 \\ +875 \\ +875 \\ \hline 2625 \end{array} \qquad \begin{array}{r} 975 \\ +975 \\ +975 \\ \hline 2925 \end{array}$$

Answer 1 is wrong because we are told the digits are non-zero and $l = 0$.

Answer 4 is wrong because we are told the digits are distinct so $l \neq f$.

Why do this activity?

This activity does not involve any mathematical knowledge beyond what is learned in primary school. It does call for learners to think creatively and mathematically. If the teacher manages the lesson well then many learners can actually have the pleasure of making a breakthrough and solving the problem for themselves.

Learning objectives

In doing this activity students will have an opportunity to develop problem solving skills and confidence in themselves as mathematicians.

Generic competences

We need to prepare children for a job market where existing knowledge and skills have limited value unless they can be applied in novel ways to produce new knowledge that solves today's complex problems to improve the quality of life for all.

In doing this activity students will have an opportunity to:

1. **think mathematically,** reason logically and give explanations and proofs;
2. **think flexibly**, be creative and innovative - to apply knowledge and skills;
3. **solve problems** – to interpret and solve problems.

Suggestions for teaching

Start with the diagnostic quiz and give sufficient time for the learners to work by themselves and check all 4 suggested solutions. If they understand the question then they will rule out answers 1 and 4 immediately and only have to check the addition in answers 2 and 3. This gives good practice in reading the question and adding up.

Then tell the learners that you are giving them this puzzle so that they can think like mathematicians and feel good when they find an easy way to solve it.

Tell them they should not do any boring adding up and that they only need primary school mathematics. They must use the hint.

Give the learners time to work on it by themselves.

To give as many learners as possible the chance to crack the problem for themselves you might check on each learner who finds a solution, praise them if they have solved the problem and then give them an extension task.

In a mixed ability class where some learners may not succeed by themselves without encouragement, talk to those learners individually and show them that you believe that they can do it. Say things like "I know you can do this one, keep trying and make sure you use the hint". Remember that brain development research has established that everyone can do maths and it is self doubt that holds people back. As a very last resort you could allow learners to work in pairs after a while – but pair two learners who are **both** struggling so that they help each other but finally know they have succeeded by their own efforts.

Finally ask some learners to explain the method of solving the problem so that they get practice in presenting and communicating mathematical ideas.

Key questions

- What reason could there by for bringing in zero?
- Which number goes with zero?
- You have to put the numbers into pairs, can you think of a **SPECIAL** way to do that?
- What is **SPECIAL** about the partnerships between the pairs of numbers?
- How many pairs will there be? Good, that is an easy number to multiply by!!
- Why is one million an odd man out.

Follow-up ideas

Target 100 Game https://aiminghigh.aimssec.ac.za/years-4-5-target-100/
Target 1000 Game https://aiminghigh.aimssec.ac.za/years-4-7-target-1000/
Half a Million Game https://aiminghigh.aimssec.ac.za/years-6-7-half-a-million-game/
Sandwiches https://aiminghigh.aimssec.ac.za/years-6-to-12-sandwiches/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **beyond** the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	**Upper Primary Age 9 to 11**	**Lower Secondary Age 11 to 14**	**Upper Secondary Age 15+**
South Africa	**Grades R and 1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
USA	**Kindergarten and G1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
UK	**Reception and Years 1 to 3**	**Years 4 to 6**	**Years 7 to 9**	**Years 10 to 13**
East Africa	**Nursery and Primary 1 to 3**	**Primary 4 to 6**	**Secondary 1 to 3**	**Secondary 4 to 6**