AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

CURIOUS NUMBER

Can you use the digits 1, 2, 3, 4, 5 and 6, each only once, to make a number that is divisible by 6 with the following properties:

the first digit from the left is divisible by 1
the number formed by the first 2 digits from the left is divisible by 2
the number formed by the first 3 digits from the left is divisible by 3
the number formed by the first 4 digits from the left is divisible by 4
the number formed by the first 5 digits from the left is divisible by 5
the number formed by all 6 digits is divisible by 6?

Help

You could work with a partner or by yourself to try to decide which of the following 4-digit numbers are solutions to the puzzle.

What do you notice about the patterns in the way the numbers are written in rows and columns. Why do you think that the numbers have been arranged in this way?

Start by crossing out all the numbers in the list that you know **cannot** be solutions:

1234, 1243, 1324, 1342, 1423, 1432

2134, 2143, 2314, 2341, 2413, 2431

3124, 3142, 3214, 3241, 3412, 3421

4123, 4132, 4213, 4231, 4312, 4321

Extension

You could attempt to order the ten digits from 0 - 9 in the same way. Either play the challenging **American Billions Game** with a friend or try this challenge as a solitaire puzzle.

You take it in turns to choose and place a card to the right of the cards that are already there.

After two cards have been placed, the two-digit number must be divisible by 2.

After three cards have been placed, the three-digit number must be divisible by 3.

After four cards have been placed, the four-digit number must be divisible by 4.

And so on!

Keep taking it in turns until one of you gets stuck.

NOTES FOR TEACHERS

SOLUTION

Notice that the sum 1+2+3+4+5+6=21 is divisible by 3, so if the six digit number ends in an even digit (so that it is also divisible by 2) it must then be divisible by 6.

There are two solutions 123654 and 321654

The fifth number has to be the 5 as the 5-digit number is a multiple of 5.
The second, fourth and sixth numbers must be even because they are multiples of even numbers. The first and third numbers could only be either 1 or 3 so there are only 12 possibilities:
123654, 123456, 143256, 143652, 163254, 163452
321654, 321456, 341256, 341652, 361254, 361452

Looking at the first three numbers they must be 123, 143, 163, 321, 341 and 361 but only 123 and 321 are divisible by 3 so this reduces the possibilities to 123654, 123456, 321654, and 321456. Checking each number gives the solutions 123654 and 321654.

Diagnostic Assessment This should take about 5–10 minutes.

Write the question on the board, say to the class:
"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers. It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

8976 divided by 12 is:

A. 897 **B.** 749 **C.** 748 **D.** None of these

The correct answer is C
Possible misconceptions:

A. Perhaps students who gave this answer have a muddled memory about dividing by 10.

B. Students who gave this answer may have used the remainder 9 after dividing 57 by 12 instead of carrying this to the units place and dividing 96 by 12.

D. Perhaps these students just guessed.

https://diagnosticquestions.com

Why do this activity?

This activity helps learners to practise division and to gain better number sense. Knowledge of divisibility rules is useful and this activity can be introduced as a number puzzle to be solved giving an obvious use for divisibility rules. Learning new ideas can best be motivated if the learner can understand a need and a use for that knowledge.

This activity presents opportunities for discussing efficient or elegant solutions and working in a systematic way. Working first on simpler cases may help students to gain insights that can then be applied to the problem itself and to see how to solve the big problem. It is good for students to learn to think about this, and other strategies for tackling problems, when they face other problems in future.

Learning objectives

In doing this activity students will have an opportunity to:

- practise number skills;
- learn and understand simple rules for divisibility.

Generic competences

In doing this activity students will have an opportunity to:

- think mathematically and work systematically;
- apply knowledge and skills;
- interpret problems and use common problem solving strategies;
- collaborate/work in a team.

Suggestions for teaching

Start with the Diagnostic Quiz and if necessary write the multiples of 12 up to 108 on the board to help learners to do the division.

Write up the number 123 (one hundred and twenty three) on the board for all to see and ask learners to talk together about anything they notice or know about the number. Share some suggestions, which might include, for example, that it is odd, it has three digits, the digits are consecutive, it is a multiple of three ...

Explain that this is in fact a special number. If you look at just the first digit (1), it is divisible by 1. The number given by just the first two digits (12) is divisible by 2. The number given by all three digits (123), is divisible by 3. Say to the class something like: "I wonder whether there are any other ways of arranging the digits 1, 2 and 3 to make a number which has these same properties?" We want to rearrange the digits 1, 2 and 3 to make a number that is divisible by 3 ... so that when the third digit is removed it becomes a two-figure number divisible by 2 ... and when the second digit is removed it becomes a one-figure number divisible by 1.

You could suggest that another way of ordering the numbers 1, 2 and 3 is, for example, 2 then 3 then 1 to make the number two hundred and thirty one (231). So, is the number 231 divisible by 3? How do they know? [Yes, it is. Some children might mentally perform a division calculation. Others might know that if the digits add to a number that is divisible by 3, then the whole number is divisible by 3.] So far, so good.

Next take off the last digit to give the number 23. Is this divisible by 2? [No – 23 is an odd number.] So, the number 231 doesn't meet all the requirements.

Then ask the class “How can you tell if a number is divisible by 5?” make a heading DIVISIBILITY RULES on the board. Establish that numbers are divisible by 5 if they end with 0 or 5 and write this rule on the board.

Then ask the class “How can you tell if a number is divisible by 2?” and establish that numbers are divisible by 2 if they end with 0, 2, 4, 6 or 8 (an even number) and write this rule on the board.

Now ask “Do you know any divisibility rules?” and go over the rules for divisibility by 3 and by 4, and by 6.

A number is divisible by 3 if its digits, when added together, are divisible by 3. For example, take 174:1+7+4=12 which is divisible by 3. You can add digits as many times as you want. 12:1+2=3

A number is divisible by 6 if it is an even number and it is divisible by 3.

A number is divisible by 4, if the tens and units form a number which is divisible by 4, for example 732 and 9048 are divisible by 4 (because 32 and 48 are divisible by 4, but 338 and 2342 are not (because 38 and 42 are not divisible by 4). (Why does this work?)

At this stage ask the class to read the question and then some students to explain in their own words what

they think it means. Invite pairs or small groups to continue working on this problem and give them a taste of what their next challenges will be: "Can they do the same for four-digit numbers using the digits 1, 2, 3 and 4?" "How about five-digit numbers using 1, 2, 3, 4 and 5?" Suggest to the class that for a six-digit number they will need to use the tests for divisibility and to make a table to keep track of where the digits 1 to 6 could go.

Once the learners have had a go at the three-digit and perhaps four-digit possibilities, bring them together to talk about their methods. Note that it is not possible to find a four-digit or a five-digit number that follow these rules but you can ask the learners to explain why to convince you this is the case. This will bring up the fact that some digits have to go in certain places. For example, which places have to have even numbers?

The discussion at the end of the lesson could involve sharing solutions to the six-digit problem but, in particular, invite the learners to comment on how having a go at simpler versions of the task helped them tackle the six-digit (or more) challenge. How do they think they might have got on if they'd have been presented with the six-digit version at the start? What does this tell us about solving mathematical problems?

Key questions

- What makes a number divisible by one/two/three/four/five/six ...?
- Where do the even numbers have to go?
- So where do the odd numbers have to go?
- Where does the five have to go?

Follow-up ideas

Target Division https://aiminghigh.aimssec.ac.za/years-5-7-target-division/

Target 4 by 2 division https://aiminghigh.aimssec.ac.za/years-6-7-target-4-by-2-division/

Magic Numbers https://aiminghigh.aimssec.ac.za/years-5-9-magic-numbers/

Biggest Divisor Same Remainder

https://aiminghigh.aimssec.ac.za/years-9-or-10-biggest-divisor-same-remainder/

American Billions Game http://nrich.maths.org/796

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **beyond** the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6