AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE
TEACHER NETWORK

Title: CUBOIDS (Grades 8 to 10)

Find a cuboid (with edges of integer values) that has a surface area of exactly 100 square units. Is there more than one?

Solution

If we use a method of trial and error we need to be systematic to avoid repeating calculations and to be sure of finding all the solutions. Label the lengths of the edges x, y, and z where x ≤ y ≤ z then the results for the surface area are given in the following table. We don't try all values, just those which give a surface area around 100. Note that for any chosen x and y then the surface area increases as we increase z so we try some values and pick the values of z accordingly. Once we get a surface area of more than 100 we don't need to try any larger values of z. **This shows that there are only 2 solutions: 1, 2 , 16 and 2, 4, 7.**

This method does not require any algebra or solving equations, just the calculation of surface areas.

x=1 y=1			**x=1 y=2**			**x=1 y=3**			**x=1 y=4**			**x=1 y=5**			**x=1 y=6**			**x=1 y=7**	
z	**S Area**		**z**	**S Area**		**z**	**S Area**		**z**	**S Area**		**z**	**S Area**		**z**	**S Area**		**z**	**S Area**
10	42		10	64		10	86		5	58		5	70		6	96		7	126
20	82		15	94		11	94		6	68		6	82		7	110			
30	122		16	100		12	102		7	78		7	94						
21	86								8	88		8	106						
22	90								9	98									
23	94								10	108									
24	98																		
25	102																		
x=2 y=2			**x=2 y=3**			**x=2 y=4**			**x=2 y=5**			**x=2 y=6**							
z	**S Area**		**z**	**S Area**		**z**	**S Area**		**z**	**S Area**		**z**	**S Area**						
10	88		5	62		4	64		5	90		6	120						
11	96		6	72		5	76		6	104									
12	104		7	82		6	88												
			8	92		7	100												
			9	102															
x=3 y=3			**x=3 y=4**			**x=3 y=5**						**x=4 y=4**			**x=4 y=5**				
z	**S Area**		**z**	**S Area**		**z**	**S Area**					**z**	**S Area**		**z**	**S Area**			
3	54		4	80		5	110					4	96		5	130			
4	66		5	94								5	112						
5	78		6	108															
6	90		7																
7	102																		

The use of algebra simplifies the calculations and speeds up the process.
Suppose we label the lengths of the edges x, y, and z where x ≤ y ≤ z then the surface area is
$2(xy + yz + zx) = 100$
So $xy + yz + zx = 50$ and $z = (50 - xy)/(x + y)$.

For each pair of values of x and y we can calculate z and we get solutions where z is a whole number. Calculations are recorded in the table below with numbers rounded to 1 decimal place. Notice that as x and y increase z decreases so we stop at the values for which z becomes less than x or y.

x = 1		x = 2		x = 3		x = 4		x = 5	
y	z	y	z	y	z	y	z	y	z
1	49/2=24.5	2	46/4=11.5	3	41/6=6.8	4	34/8=4.25	5	25/10=2.5
2	48/3=16	3	44/5=8.8	4	38/7=5.4	5	30/9=3.3	6	
3	47/4=11.75	4	42/6=7	5	35/8=4.4	6		7	
4	46/5=9.2	5	40/7=5.7	6		7		8	
5	45/6=7.5	6	38/8=4.75	7		8			
6	44/7=6.3	7		8					
7	43/8=5.4	8							

Checking: 2(2 + 16 + 32) = 100 and 2(8 + 14 + 28) = 100
This shows that the only two solutions are 1, 2, 16 and 2, 4, 7.

Notes for teachers

Why do this activity?

This activity requires a lot of calculations of surface areas, within a rich problem solving context, where it is necessary to work systematically to record the values which have been tried to avoid repeating calculations and to be sure of finding all the solutions.

Possible approach

Work with a specific cuboid, eg 2×3×5, or a breakfast cereal box, to establish how to calculate surface area of cuboids. Learners could practise working out surface area mentally on some small cuboids.

Present the problem and ask learners to keep a record of things that they tried that didn't work (and what was wrong) as well as things that did work. In this initial working session, try to ensure that learners are calculating surface area correctly. Don't give them either of the tables above to fill in. It is much better for learners to experiment with their own ways of recording and, later in the lesson, for the whole class to discuss the different ways that different learners have recorded their results and the advantages and disadvantages of the different methods.

This might be a good lesson in which to allocate five minutes at the end to ask students to reflect on what they have achieved, which methods and ideas were most useful, and what aspects of the problem remain unanswered.

Key questions

- Have you found none/one/some or all of the solutions
- Is there a cube that will work?
- How might you organise a systematic search for the cuboids with surface area 100?

Possible extension

The main extension activity could focus on the convincing argument that all solutions have been found. Once this has been answered, you might like to consider these extensions:

- Express the method for calculating surface area, algebraically.
- What surface area values will generate lots of cuboids and which give none or just one?
- Could you set up a spreadsheet to help with the calculations?

Possible support

In groups, or as a class, keep a record of all cuboids whose surface areas have been calculated.
Award ten points for a bulls eye "100", five points for each 95–105, and two points for 90–110.
Any miscalculated results could lose points, providing motivation for peer checking, and helping each other.