AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
TEACHER NETWORK

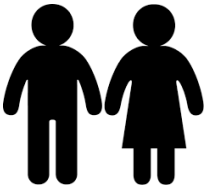
COUPLES PUZZLE

In a small village two thirds of the adult men are married to three quarters of the adult women.

How big is the smallest village for which this is true?

Can you describe the populations of other villages for which this is true?

HELP



Cut out some of these figures (see below). Arrange them in pairs and singly to fit the conditions of the question.

Try a few numbers and see if you can find the answer.

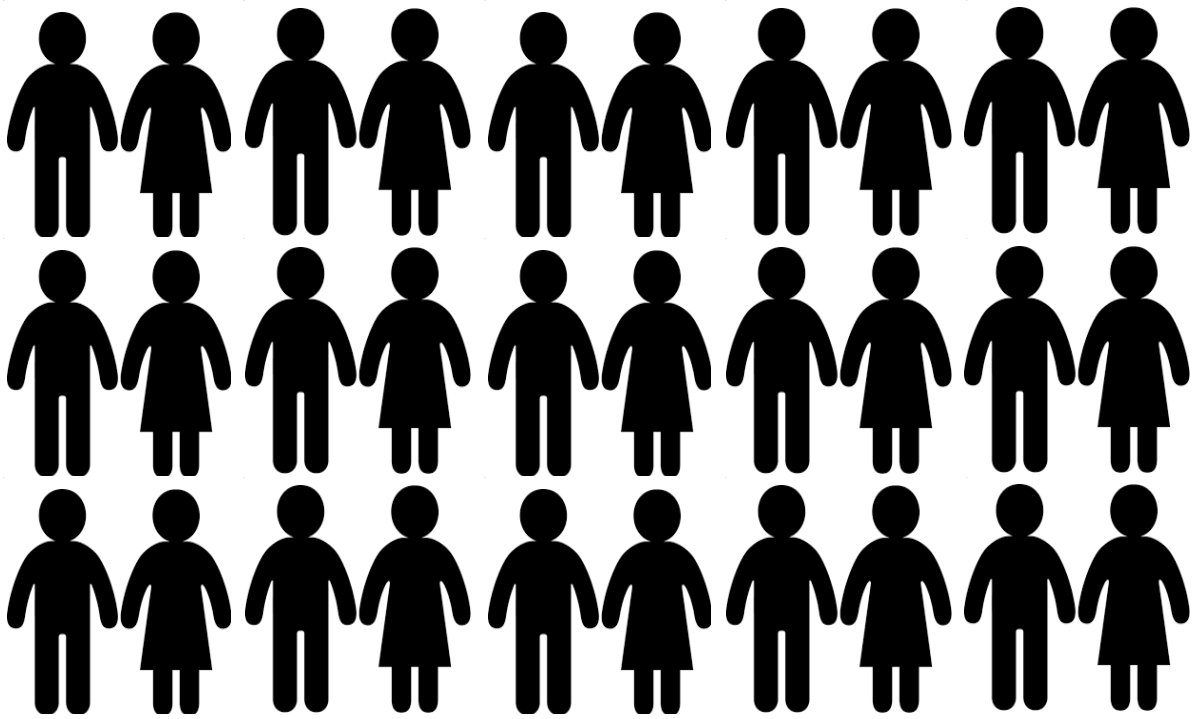
What about 3 couples, or 4 couples or 5 couples…? Why don't these numbers for 3, 4 and 5 couples work?

How many men in the village if two thirds of the adult men are married?

How many women in the village if three quarters of the adult women are married?

NEXT

Make up your own example with different numbers and swap it with a friend so you have to solve each other's puzzles.



NOTES FOR TEACHERS

SOLUTION

Method 1 Multiples

The number of adult men must be a multiple of 3 so it could be 3, 6, 9, 12, 15, … and so on (otherwise you can't find thirds that are whole numbers) so there could be 2, 4, 6, 8, 10,… married men.

Similarly, the number of adult women must be a multiple of 4 so it could be 4, 8, 12, 16 … and so on and three quarters of the women would then be 3, 6, 9, 12, …

So, in the smallest village there are 9 men and 8 women with 6 married couples, that is 17 adults all together.

Method 2 Fractions

$\frac{2}{3}=\frac{4}{6}=\frac{6}{9}=\frac{8}{12},\ldots$

$\frac{3}{4}=\frac{6}{8}=\frac{9}{12}=\frac{12}{15},\ldots$

From the numerators we see that the smallest values that are equal are 6 representing 6 out of 9 men (⅔ of the adult men) and 6 out of 8 women (¾ of the adult women).
From the denominators we get the total number of adults in the group, that is 9 + 8 = 17.

Method 3 Algebra

If there are x men and y women then $\frac{2}{3}x = \frac{3}{4}y$ so $8x = 9y$. The smallest numbers for which this can be true is $x = 9$ and $y = 8$. This gives 6 married couples and tells us that the group size is: $x + y = 17$.

Larger villages would have a multiple of 6 couples, for example:

12 couples with 18 men and 16 women making 34 adults,

18 couples with 27 men and 24 women making 51 adults etc.

Why do this activity?

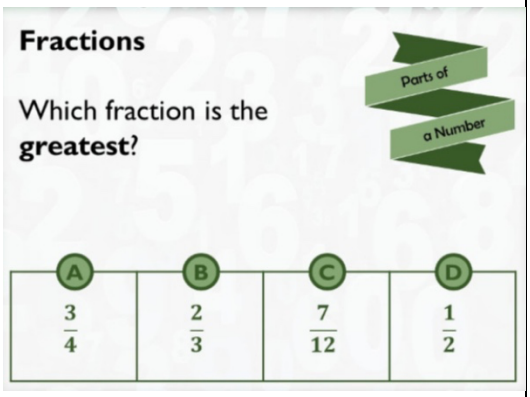
Learners need to be able to solve non-standard problems and it is much harder to solve problems using recently learned mathematical concepts. This activity requires no mathematical knowledge other than either an understanding of fractions, or an understanding of multiples. The 'real life' context gives the problem a grounded (concrete) quality. The activity gives learners practice in problem solving and it gives them the opportunity to devise their own methods and to develop their own explanations.

Learning objectives

In doing this activity students will have an opportunity to review earlier work on fractions and multiples and to develop problem solving skills.

Diagnostic Assessment This should take about 5–10 minutes.

Write the question on the board, say to the class:
"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".



1. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
3. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
4. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

The correct answer is A. because $\frac{3}{4} = \frac{9}{12}$.

Possible misconceptions:

Students giving answers **B., C.** and **D.** do not understand how to find equivalent fractions with denominator 12. https://diagnosticquestions.com

Generic competences

We need to prepare children for a job market where existing knowledge and skills have limited value unless they can be applied in novel ways to produce new knowledge that solves today's complex problems to improve the quality of life for all.

In doing this activity students will have an opportunity to:

- think mathematically and to reason logically;
- apply knowledge and skills;
- solve and interpret problems.

Suggestions for teaching

Write the problem on the board and ask the learners to work in pairs to solve it. Circulate around the class observing the methods they are using and, if necessary, asking Key Questions. If they find answers then tell them to make sure that they have a good explanation to give to the class.

As always with the AIMING HIGH material, if learners are struggling give them the HELP strip and if they finish before the rest of the class, give them the NEXT strip.

After they have had sufficient time for most pairs to reach an answer tell the class to work in groups of 4 to discuss their methods. Have they used the same method as the other pair in their group?

Finally ask learners to explain their methods to the class.

Review and summarize facts they should know about multiples and fractions.

You could suggest using algebra as an extension for groups who have finished before the rest of the class. At the end, if none of the learners have given explanations using algebra you might give the whole class a few minutes to try to use algebra and then discuss that method as a class.

Key questions

- Could there be 10 men in the village? Why or why not?
- Could there be 10 women in the village? Why or why not?
- If each husband only has one wife, what do you know about the number of married men and married women?
- So can you make a list some of the possible numbers for all the men?
- So can you make a list some of the possible numbers for all the women?
- Can you write down some equivalent fractions to ⅔?
- What about writing down some equivalent fractions to ¾?
- What do those numerators tell you?
- What do those denominators tell you?
- Can you find the same answers using a different method?

Follow up

This puzzle is about fractions, factors and multiples. The class could gain a deeper understanding of factors and multiples with
https://aiminghigh.aimssec.ac.za/years-6-12-factors-and-multiples-game/
On the AIMING HIGH website (which you can access on the AIMSSEC App) there are many Learning Packs of Fractions. Just try a Topic Search.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13