

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES SCHOOLS ENRICHMENT CENTRE (AIMSSEC)

AIMING HIGH

BIGGEST DIVISOR SAME REMAINDER



What is the largest number that, when divided into each of 1905, 2587, 3951, 7020 and 8725, leaves the same remainder every time?

When a problem seems difficult a good strategy is to try a simpler case.

The diagram should help you to find the answer to the same question for 79, 35 and 57 and it should give you a hint for the method to use for the more difficult problem.

HELP

An even simpler case you could work on is to do the same problem for the three numbers 20, 38 and 50.

This is much easier as the numbers are smaller and you have to work on 3 numbers rather than 5 numbers.

Once you have succeeded with the two simple cases go back to the big challenge.

See also 'Spot The Mistake' to help you to review the operation of division: <u>https://aiminghigh.aimssec.ac.za/years-4-7-spot-the-mistake/</u>

NEXT

Make up your own similar problems and exchange them with a partner so that you solve each other's problem.

The 'Magic Numbers' activity also involves division: https://aiminghigh.aimssec.ac.za/years-5-9-magic-numbers/



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NOTES FOR TEACHERS

SOLUTION

Looking at 79, 35 and 57 we see that they can all be divided by 11 with remainder 2. The diagram shows $7 \times 11+2$, $3 \times 11+2$ and $5 \times 11+2$. The key to finding the solution is to notice that the differences between the numbers (79-35=44, 79-57=22 and 57-35=22 in this case) have a COMMON FACTOR because one remainder is subtracted from the other,

To solve the big problem we have to find the biggest common factor of the differences between the 5 numbers: 1705, 3069, 1364, 682 and 6820.

To do this factorize the differences: $1705 = 5 \times 341 = 5 \times 11 \times 31$ $3069 = 3 \times 1023 = 3 \times 3 \times 341 = 3^2 \times 11 \times 31$ $1364 = 2 \times 682 = 2 \times 2 \times 341 = 2^2 \times 341$

So it looks as if 341 may be the answer. To check this:

 $682 = 2 \times 341$ $6820 = 2^2 \times 5 \times 341$

The largest number which when divided into each of 1905, 2587, 3951, 7020 and 8725, leaves the same remainder each time is 341 and the remainder is 200 each time.

Diagnostic Assessment This should take about 5–10 minutes.

- 1. Write the question on the board, say to the class:
- "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer
- 2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
- 3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
- 4. Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers. It is important for learners to explain the reason for their answer otherwise many learners will just make a guess. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.
 The correct answer is R

r		1 The correct answer is D
Work out:		Possible misconceptions:
		A. Students could be confused about which is divisor and which is dividend.
	642 ÷ 8	C . Students may have written the remainder 2 in the tenths place (a common error) or
		may have just stopped there and not worked out the hundredth digit in the quotient.
		D . This is correct. Some students may know only about remainders and not how to
		carry out division to get the decimal part of the answer.
A = 130.2		https://diagnosticquestions.com
B = 80.25		
C = 80.2		
D = 80 r2		
1		

Why do this activity?

This problem solving activity requires only an understanding of the operation of division and finding prime factors, so these ideas are reviewed as part of the work to solve the problem. This activity helps students to develop problem solving skills. It is a good example of trying a simpler case helping to solve a problem.



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Learning objectives

In doing this activity students will have an opportunity to:

- to review prime factorization and the operation of division;
- develop problem solving skills.

Generic competences

In doing this activity students will have an opportunity to:

solve and interpret problems in a variety of situations

- to communicate in writing and speaking according to the audience;
- o communicate, exchange ideas, criticise, and present information and ideas to others.

Suggestions for teaching

You may like to use 'Target 4 by 2 Division' as a lesson starter game:

https://aiminghigh.aimssec.ac.za/years-6-7-target-4-by-2-division/

In order for learners to practice reading and interpreting questions you could give the learners this question together with the hint to solve first the simpler case of 79, 35 and 57 without giving any advice about how to proceed. Let the learners work in pairs for long enough for many pairs to have solved the simpler case and started on the big problem and perhaps some learners to have solved that.

Then invite learners to explain how they solved the simpler problem. It may be helpful to have several learners give the explanation so that everyone in the class understands that the key to finding the solution is to notice that the differences between the numbers has a COMMON FACTOR because one remainder is subtracted from the other when you find the difference.

Finally you can summarise what has been learned including the meanings of the words divisor, factor and remainder.

Key questions

- How does the simple case diagram help? What do you see in it?
- What are the differences between the numbers in the simple case? What does that tell you?
- What happens to the remainder when you subtract one number from another?
- Can you find the factors of those numbers?

Follow up

Spot The Mistake https://aiminghigh.aimssec.ac.za/years-4-7-spot-the-mistake/

Magic Numbers https://aiminghigh.aimssec.ac.za/years-5-9-magic-numbers/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6. For resources for teaching A level mathematics see https://nrich.maths.org/12339 Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA. Lower Primary **Upper Primary** Lower Secondary **Upper Secondary** or Foundation Phase Age 5 to 9 Age 9 to 11 Age 11 to 14 Age 15+ **South Africa** Grades R and 1 to 3 Grades 10 to 12 Grades 4 to 6 Grades 7 to 9 USA Kindergarten and G1 to 3 Grades 7 to 9 Grades 10 to 12 Grades 4 to 6 Years 7 to 9 UK **Reception and Years 1 to 3** Years 4 to 6 Years 10 to 13 **East Africa** Nursery and Primary 1 to 3 Primary 4 to 6 Secondary 1 to 3 Secondary 4 to 6