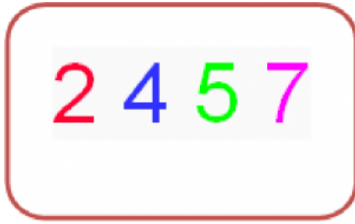## AND SO ON AND SO ON

$2\ 4\ 5\ 7$

If you change the order of these digits you get many different numbers, for example 2547, 2574, 2745, … Write down all the four digit numbers made by using each of the digits 2, 4, 5, 7 once.

How many of these numbers have 2 in the thousands place? How many have 2 in the hundreds place? How many have 2 in the tens place? How many have 2 in the units place?

Now find what these 24 four digit numbers add up to.
How many methods can you find for solving this problem?

Help

You could first work on this simpler example:
How many numbers can you find using the digits 1 and 2? What do they add up to?
Answer 12 and 21 adding up to: $10 + 1 + 20 + 2 = 33$.

How many numbers can you find using the digits 1, 2 and 3? What do they add up to?
Answer 312, 132, 123, 321, 231, 213 putting 3 as the 1st digit in the numbers above, then the 2nd then the 3rd
$(300 + 10 + 2) + (100 + 30 + 2) + (100 + 20 + 3) + (300 + 20 + 1) + (200 + 30 + 1) + (200 + 10 + 3)$

$= 3(300 + 200 + 100) + 3(30 + 20 + 10) + 3(3+2+1)$

$= 1800 + 180 + 18$

$= 1998$

Now try for yourself: How many numbers can you find using the digits 1, 2, 3 and 4? What do they add up to?

Extension

How many 4 digit numbers can you find using the digits 1, 7, 2 and 6? What do they add up to?

Also see: https://aiminghigh.aimssec.ac.za/grades-6-to-10-magic-13837/

https://aiminghigh.aimssec.ac.za/years-7-to-9-writ-large/

NOTES FOR TEACHERS

SOLUTION

First Method

There are 24 different numbers. We can add these to get 119988.

2457 2475 2547 2574 2745 2754
4257 4275 4527 4572 4725 4752
5247 5274 5427 5472 5724 5742
7245 7254 7425 7452 7524 7542

Second Method PLACE VALUE

Each digit appears six times in each position so 2000 occurs 6 times, 200 occurs 6 times, 20 occurs 6 times and 2 occurs 6 times and the same for each digit. So the total is given by:

6 x (2000 + 200 + 20 + 2) = 13332	6 x (2000 + 4000 + 5000 + 7000)= 6 x 18000
6 x (4000 + 400 + 40 + 4) = 26664	6 x (200 + 400 + 500 + 700) = 6 x 1800
6 x (5000 + 500 + 50 + 5) = 33330	6 x (20 + 40 + 50 + 70) = 6 x 180
6 x (7000 + 700 + 70 + 7) = 46662	6 x (2 + 4 + 5 + 7) = 6 x 18
TOTAL 119988	TOTAL 119988.

Third Method

There are 24 different numbers and we can look at them in pairs that switch the 2 and 7 and the 5 and 4 so that the pair adds up to 9999.

Example 2457, 7542 and 2475, 7524, and all the numbers on the top and bottom row above.
Similarly 4257, 5742 and 4275, 5724…

So the sum is 12 x 9999 = 119988

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

Place Value

Introducing
Percentages

What is the **place value** of the 7 in the number 4275931?

A	B	C	D
Seven thousand	Seven million	Seventy thousand	Seven thoudandths

The correct answer is C
Possible misconceptions:

A, B and **C** Students giving these answers have a poor understanding of place value or the associated language, or they just guessed.

https://diagnosticquestions.com

Why do this activity?

For younger learners this activity can give them practice using place value, adding 4 digit numbers and multiplying 4 digit numbers by a single digit.

For older learners the activity can focus on looking for patterns and finding all possible arrangements. It will help them to develop problem solving skills. Working on this problem also gives learners an experience of the advantage of working systematically.

Learning objectives

In doing this activity students will have an opportunity to:
o   gain experience of place value, mathematical thinking and addition and multiplication of 4 digit whole numbers;
o   develop problem solving skills in particular looking for relationships between numbers, including patterns.

Generic competences

In doing this activity students will have an opportunity to:
o   solve problems – to solve and interpret problems in a variety of situations
o   to communicate in writing and speaking according to the audience:
    •   communicate, exchange ideas, criticise, and present information and ideas to others
    •   analyze, reason and record ideas effectively

Suggestions for teaching

Start with the diagnostic quiz and, if necessary, a review of place value.

You might start the lesson with this game:
https://aiminghigh.aimssec.ac.za/grades-5-to-7-target-10-thousand/

Teachers might ask different groups of students to work on different methods, and to write up an account of their solutions either in their notebooks if working in pairs, or on a poster if working in larger groups. Then students could be asked to give presentations on their methods to the whole class so that everyone sees all the methods.

Method 2 PLACE VALUE
Questions for independent work by individuals or in pairs or small groups:
1.   How many thousands are there in the number 2457?
2.   Using the same digits but different numbers of hundreds, tens and units, list all the numbers that have two thousands.
3.   Do the same listing all the numbers with 4 thousands, all the numbers with 5 thousands and all the numbers with 7 thousands. Write these numbers in a list so that you can easily check that you have listed all the possibilities.
4.   Explain why there are 6 and only 6 arrangements for each set making 24 in all.
5.   Do the same with the digits 2457 to list all the numbers with 4 thousands. Then with 5 thousands, then with 7 thousands (6 of each)
6.   How many times do we see 2000?
     How many times do we see 200?
     How many times do we see 20?
     How many times do we see 2?
     Now add these numbers to get 13332. Do the same with the other sets of numbers and find the final total.

Suggestions for a class lesson
Start with the number 2457. Ask the class how many thousands are there in the number? Can they think of another number using the same digits with two thousand but different numbers of hundreds, tens and units.

List the numbers suggested by the learners in a row on the board. Ask "what would be a good order to write the numbers in so that we know we have listed all the possibilities? Ask the learners to explain why there are 6 and only 6 arrangements for each set making 24 in all.

Do the same with the digits 2457 to list all the numbers with 4 thousands. Then with 5 thousands, then with 7 thousands (6 of each)

Then ask "how many times do we see 2000?" (Answer 6). "How many times do we see 200?" "How many times do we see 20?" "How many times do we see 2?" Then add these numbers to get 13332. Do the same with the other sets of numbers.

Method 3 USING PATTERNS OF PAIRS ADDING UP TO 9999

Questions for independent work by individuals or in pairs or small groups:
1. How many thousands are there in the number 2457?
2. Using the same digits but different numbers of hundreds, tens and units, list all the numbers that have two thousands.
3. Do the same with the digits 2457 to list all the numbers with 4 thousands, then with 5 thousands, then with 7 thousands (6 of each). Write these numbers in a list so that you can easily check that you have listed all the possibilities.
4. Explain why there are 6 and only 6 arrangements for each set making 24 in all.
5. List the pairs of numbers with the 2 and 7 digits interchanged and the 4 and 5 digits in the same place. What do these pairs of numbers add up to?
6. List the pairs of numbers with the 4 and 5 digits interchanged and the 2 and 7 digits in the same place. What do these pairs of numbers add up to?
7. Can you now easily find the final total?

Suggestions for a class lesson
Alternatively after listing the 6 numbers starting with 2, suggest that the class think about numbers with 7 thousand and when a number is suggested write it under the corresponding number already on the board with 7 digit under the 2, the 2 digit under the 7. The 5 digit under the 4 and the 4 digit under the 5. Ask "Can anyone see what I am doing?" Ask the class to add up the numbers that are underneath each other.

Then repeat this process with 5 thousands and the four thousands. With younger learners you might do this as a class activity. Older learners might be asked to do this individually or working in pairs.

Then review this method. What have we been doing? We have been finding pairs of numbers that add up to 9999? How many pairs? … Ask learners to explain some of the steps in this process. Finally the whole class should have a list of 12 pairs of numbers, each pair adding up to 9999. They can then be asked to work out the total of all 24 numbers.

Key questions
- Are you sure that you have listed all the possibilities? How do you know?
- How many times do we get 2000 in this list? Or 4000? 5000? 7000?
- How many times do we get 200 in this list? Or 400? 500? 700?
- How many times do we get 20 in this list? Or 4? 5? 7?
- How many times do we get 2 in this list? Or 4? 5? 7?
- How many thousands is that altogether?
- Do you notice anything about the numbers you get if you switch the 2 and 7?
- Do you notice anything about the numbers you get if you switch the 4 and 5?

Follow up
Also see: https://aiminghigh.aimssec.ac.za/grades-6-to-10-magic-13837/

https://aiminghigh.aimssec.ac.za/years-7-to-9-writ-large/

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **beyond** the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	**Grades R and 1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
USA	**Kindergarten and G1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
UK	**Reception and Years 1 to 3**	**Years 4 to 6**	**Years 7 to 9**	**Years 10 to 13**
East Africa	**Nursery and Primary 1 to 3**	**Primary 4 to 6**	**Secondary 1 to 3**	**Secondary 4 to 6**