ROD MEASURES

The image shows 10 = 9 + 4 - 3.

Using 3 rods of lengths from 1 to 10 units, and not using any rod more than once, you can measure all the lengths in whole units from 1 to 10 units.

For example, with rods of lengths 3, 4, and 9 the measurements are:
1 = 4 – 3
2 = 9 – 4 - 3
3
4
5 = 9 - 4
6 = 9 – 3;
7 = 3 + 4
8 = 9 + 3 - 4
9
10 = 9 + 4 - 3

How many ways can you find to do all these measurements with 3 rods? To be sure that you have found ALL the solutions you need to work systematically.

Do this as a group project if possible to share the work because there are many solutions.

Using 3 rods of ANY integer lengths, what is the greatest length N for which you can measure all lengths from 1 to N units inclusive? Can you beat 10 units?

What is the greatest length that can be measured using 4 rods in this way?

HELP

It would help to cut the 10 rods of lengths 1 to 10 from scrap cardboard, and to use these strips in a similar way to that shown in the image to make the different lengths.

There are many possibilities. If you are finding it difficult to get started then have a go with 2, 3 and 7. When you have made the lengths from 1 to 10 with these three lengths and recorded your results you should look for another set of three rods that will give all the lengths.

NEXT

Can you find all the lengths 1 to 13 using 3 rods?

What about four rods?

Which combinations work/do not work and why?

GUIDE FOR HOME LEARNING

RESOURCES Rods of lengths 1, 2, 3, …10 units marked into units. For example you could use unit cubes or lego. Alternatively, make paper or cardboard strips of lengths 1 to 10 and mark into units.

SOLUTION

There are 9 different sets of 3 lengths that can be combined to give the totals 1 to 10 as in the table below.

Some of these totals can be found in more than one way – can you find different ways?

The longest run using 3 lengths is from 1 to 13 with lengths 1, 3 and 9.

	1, 2, 7	1, 3, 6	1, 3, 7	1, 3, 8	**1, 3, 9**	1, 6, 9	2, 3, 7	2, 3, 9	3, 4, 9
1	1	1	1	1	**1**	1	3-2	3-2	4-3
2	2	3-1	3-1	3-1	**3-1**	9-6-1	2	2	9-4-3
3	1+2	3	3	3	**3**	9-6	3	3	3
4	7-1-2	3+1	3+1	3+1	**3+1**	9-6+1	7-3	9-2-3	4
5	7-2	6-1	7+1-3	8-3	**9-3-1**	6-5	7-2	2+3	9-4
6	7-1	6	7-1	8-3+1	**9-3**	6	7+2-3	9-3	9-3
7	7	6+1	7	8-1	**9+1-3**	6+1	7	9-2	3+4
8	7+1	6+3-1	7+1	8	**9-1**	9-1	7+3-2	9-3+2	9+3-4
9	7+2	6+3	7+3-1	8+1	**9**	9	7+2	9	9
10	7+1+2	6+3+1	7+3	8+3-1	**9+1**	9+1	7+3	9+3-2	9+4-3
11			7+3+1	8+3	**9+3-1**			9+2	
12				8+3+1	**9+3**			9+3	
13					**9+3+1**				

With 4 numbers {1, 3, 9, 27} it is possible to get totals 1, 2, 3, … 40.

From the table above we have **1, 2, 3, … 13** using only 1, 3 and 9 so by subtracting each of these from 27 we get

27 - 13 = **14**; 27 – 12 = **15**; 27 – 11 = **16**; … 27 – 1 = **26** and we can also have **27** on its own.

Then add to 27 the combinations of 1, 3 and 9 from the table that give the totals of 1, 2, 3, …13 to get

27 + 1 = **28;** 27 + 2 = **29**; … 27 + 13 = **40.**

Notice that 1, 3, 9 and 27 are powers of 3.

With 5 numbers the largest total in the sequence is 121 with the numbers 1, 3, 9, 27 and 81.

Why do this activity?

This is a good activity for developing number awareness, reinforcing the addition and subtraction rules and helping learners to remember the number bonds. It also gives learners an experience of working systematically, a skill that is very important in doing mathematics.

This task could be converted to a sequence of 'lesson starters' where each day you could suggest 3 numbers and the learners have to write down the calculations to get all the numbers from 1 to 10.

The activity could then be extended to give more lesson starters involving 4 numbers.

Learning objectives

In doing this activity students will have an opportunity to improve number sense and the ability to perform and check written and mental calculations of whole numbers including building up and breaking down numbers and using addition and subtraction as inverse operations.

Generic competences

In doing this activity students will have an opportunity to:

- **think mathematically and flexibly**, reason logically and give explanations;
- **visualize** and develop the skill of interpreting and creating visual images to represent concepts and situations;
- search systematically for solutions to a problem;
- show perseverance in pursuing a problem until every possible case has been investigated;
- work in a team to share the work and complete a task.

Diagnostic Assessment This should take about 5–10 minutes.

Show the question to the learners and say that you will give them 5 minutes to find the answer. Then tell them to: "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".

Rahid has a large number of cubic building blocks. Each block has sides of length 4 cm, 6 cm or 10 cm. Rahid makes little towers built from three blocks stacked on top of each other. How many different heights of tower can he make?

A 8 B 9 C 12 D 27

2.Notice how the learners respond. Ask all the learners to explain their answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.

3.It is important for learners to explain the reason for their answer so that they clarify their own thinking and develop their communication skills

5. Ask the learners again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answer.

6.The concept is needed for the lesson to follow, so explain the right answer or give a remedial task.

The correct answer is **B**:

10 + 10 + 10 = 30
10 + 10 + 6 = 26
10 + 10 + 4 = 24
10 + 6 + 6 = 22
10 + 6 + 6 = 20
10 + 4 + 4 = 18

6 + 6 + 6 =18
6 + 6 + 4 = 16
6 + 4 + 4 = 14

4 + 4 + 4 = 12

https://diagnosticquestions.com

Suggestions for Home Learning

RESOURCES You need 10 rods of lengths 1 to 10 marked into unit lengths. Make your rods from interlinking blocks or lego if you have it. Or make the rods from scrap cardboard and mark the unit lengths.

Young children

Mainly talking here for 5 and 6 year olds, writing it down only when they are ready. Count the units. Arrange the rods in order. Place 2 rods end to end, how long is it now? Talk about counting ON or adding, for example 8 + 5 = 13. Now do counting BACK or 'subtract' with 2 rods. (**Don't say 'take away** here' because it is all about COUNTING one way for addition, and then back for subtraction).

Then use 3 rods:  9 add 4 subtract 3 = 9 + 4 -3 = 10.

Upper Primary
All the above with **writing it down**. Gradually progress to taking 2 rods and finding ALL the totals you make (easy) then do the same for 3 rods (it's not quite so easy to find them all). Can they **explain how they know** that there are not any other combinations that they have failed to find?

Lower Secondary
All the above but now switching from manipulating the rods to **recording by writing down totals of numbers.** Now work **systematically** to gather together all **the different ways you can make different totals** with each set of rods, and to **keep a clear record of the work.** If possible, share the work with a group **so all the members of the group can contribute** to recording their answers on a chart. **Make a poster!** Put a large sheet of paper (flip chart paper, gift wrap) on the wall and record the learner's solutions as they find new ones. This might take several days. If you praise the learners when they find new solutions it might motivate them to search for solutions in their spare time.

Years 9 and 10
Now the problem is finding continuous patterns, using 4 rods – finding sets of rods where you can make EVERY length from 1 to n with **no lengths left out**. Compare this with weighing on an old-fashioned set of scales where you put the weights in ONE of the scale pans and stuff you want to weigh in the other pan. What sets of weights do you actually need?

Make more posters: For 4 numbers you might give the group 1, 3, 4 and 17 and ask them how far they can go. With these numbers it is possible to make **all the totals** from 1 to 25.

Years 11, 12 and 13
See https://aiminghigh.aimssec.ac.za/yeats-10-13-balance-power/

You can weigh all integer masses from 1 to 60 with 6 weights putting the weights in one pan and the object in the other pan. Which weights are used?
What about weighing all integer masses from 1 to 1000?
What about from 1 to n?

What could this have to do with the way computers work?
You do not have to know about binary numbers to solve this problem but for a deep understanding you might make the connection with 'on and off', 'positive and negative', 'zero and one', 'base 2 or binary numbers'.

For a **bigger challenge** can you find how many weights are needed **if you put weights in both pans**? This one is about base 3 numbers.

KEY QUESTIONS

*These are questions you ask to help learners to **think for themselves**. **STOP yourself** telling children what to do next. Instead ask a **KEY QUESTION**.*

- What have you tried?
- How did you get to this solution?
- Can you find that total in a different way?
- I see you've **not** got a 9 when using 2, 3 and 5. Can you explain that?
- Right so 2, 3 and 5 will not give you 9 so it's not a solution. Can you suggest another 3 numbers to try?

Suggestions for a Home Learning lesson for ages 10 to 12

Start with the Diagnostic Quiz. Give the learners sufficient time to write down as many solutions as they can think of. Encourage the learners to give answers rather than telling them the answers.

Start off in a very practical way with suitable rods. If you have not got rods you could cut paper or cardboard strips of lengths 1 to 10. Give the learners 3 rods and ask them to make different arrangements to make the numbers 1 to 10 like 9 + 4 – 3 = 10 as shown in the diagram.

For most learners it would be difficult to sustain a search for three numbers that can be combined to give every number from 1 to 10. So you might start by suggesting learners use **1, 2 and 7** to make all the numbers from 1 to 10. Record (write down) the answers).

1
2
3 = 1 + 2
4 = 7 – 1 – 2
5 = 7 – 2 and so on …

and ask "who can be the first to make 4?" Well done …
"who can make 5?"… Very good!

Continue in this way listing the totals from 1 to 10 so the learners have an example of the way to record their solutions.

Then tell the learners to use **1, 3 and 7** to make all the numbers from 1 to 10 and see if they can get any further working in pairs or small groups and recording their findings as above.

At this stage perhaps tell the learners that there are 9 possible sets of 3 numbers that they can do this with. Challenge them to search for the other 7 solutions (triples that can be used to make all the numbers from 1 to 10).

Advise the learners to make a note of what they have tried so they don't waste effort trying something a second time that they have already shown will not work.

If all this goes well you could challenge the learners to find a set of 4 numbers that will make all the numbers up to a total greater than 25. Who can go the highest?

Follow up

Addition puzzle that uses place value: https://aiminghigh.aimssec.ac.za/years-6-7-find-the-numbers/
A similar puzzle to Rod Measures: https://aiminghigh.aimssec.ac.za/years-6-10-special-sums/