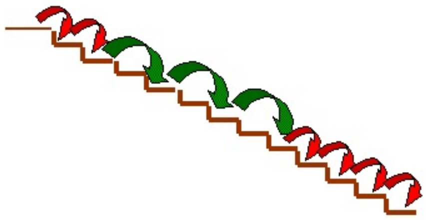
AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

This INCLUSION AND HOME LEARNING GUIDE
suggests related learning activities for all ages from 4 to 18
on the theme of PATTERNS AND SEQUENCES
Just choose whatever seems suitable for your group of learners

The original ONE STEP TWO STEPS activity was designed for Years 7 to 10

ONE STEP TWO STEPS

A staircase has 12 stairs. You can go down the stairs one at a time or two at time.



For example: You could go down 1 step (as shown in red) then one step, then 2 steps (as shown in green) then 2, 2, 1, 1, 1, 1 steps at a time as in the picture.

In how many different ways can you go down the 12 stairs, taking one or two steps at a time?

Hint: Do the question for a smaller number of stairs and see if you can find a pattern.
How many ways can you go down 3 stairs?
What about 4 stairs?
What about 5 stairs ...?

HELP

Learners should work on their own to find as many arrangements as they can with 6 stairs, then compare their answers with a partner to see if either of them has answers the other has not found.

Then each pair should compare answers with another pair. Finally, the class should list all the answers that they have found between them and try to list them in a systematic order.

NEXT

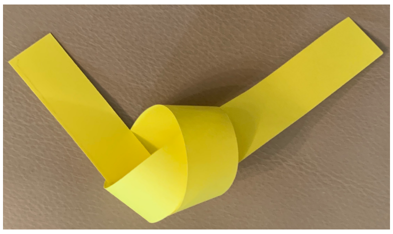
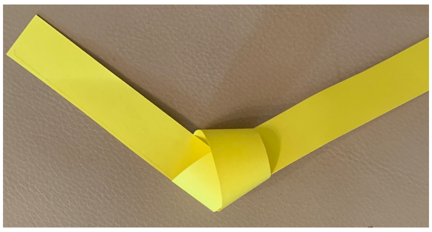
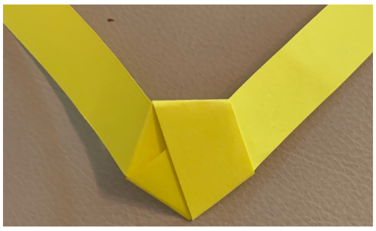
n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15...
S_n	1	1	2	3	5	8	13	21	34	55	89	144	233	377	610...

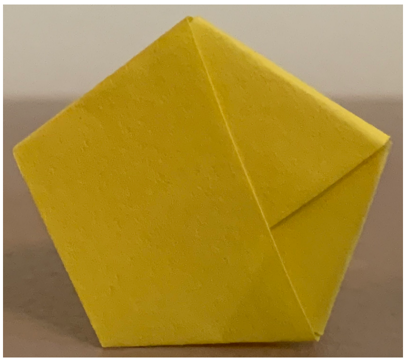
Start from $S_3 = 2$ and list every 3rd number in the sequence. What do you notice?
Start from $S_4 = 3$ and list every 4th number in the sequence. What do you notice?
Start from $S_5 = 5$ and list every 5th number in the sequence. What do you notice?

INCLUSION AND HOME LEARNING GUIDE
THEME: PATTERNS AND SEQUENCES

Early Years and Lower Primary – FINDING FIVES

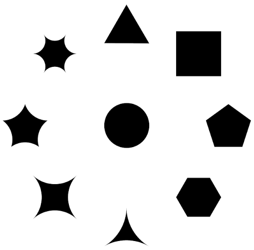
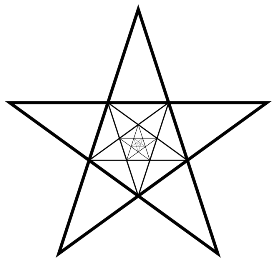
Take a strip of paper. Tie it in a knot. Gently pull the knot to close it up and then flatten it out. What do you get?

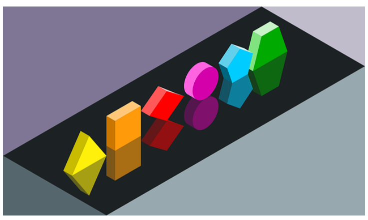




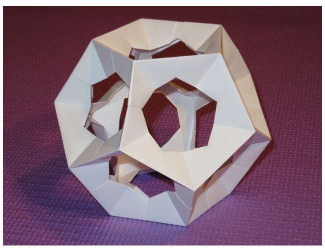
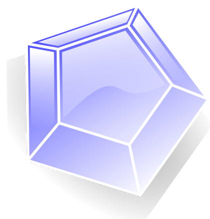


This shape is called a pentagon. Count the edges.
Where can you find fives?
Look at your hands and feet.
Stand like the picture to make a pentagon shape from the top of your head, to your hands, to your feet.
Are those lines different lengths or the same?

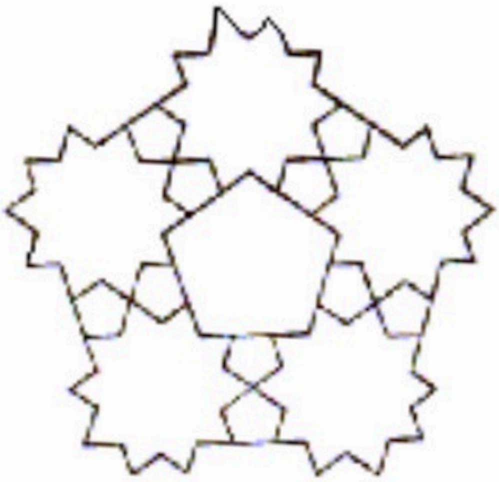
Spot the pentagons and pentagon stars in the pictures below.

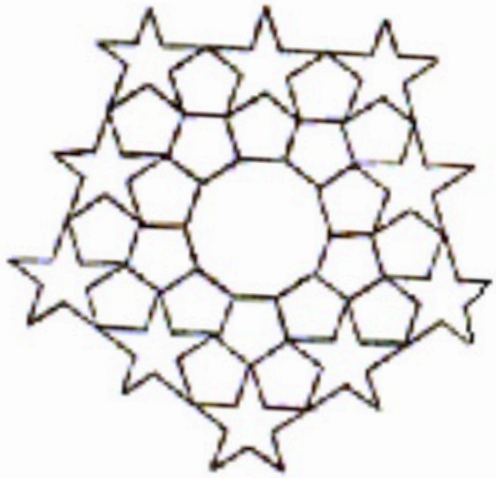




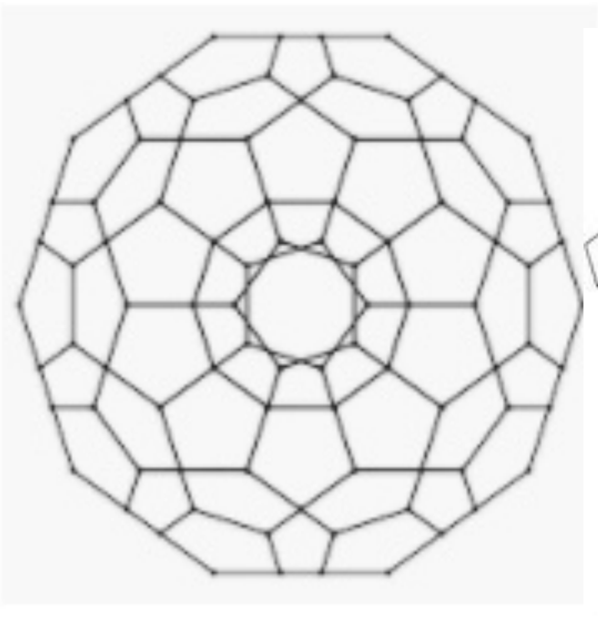


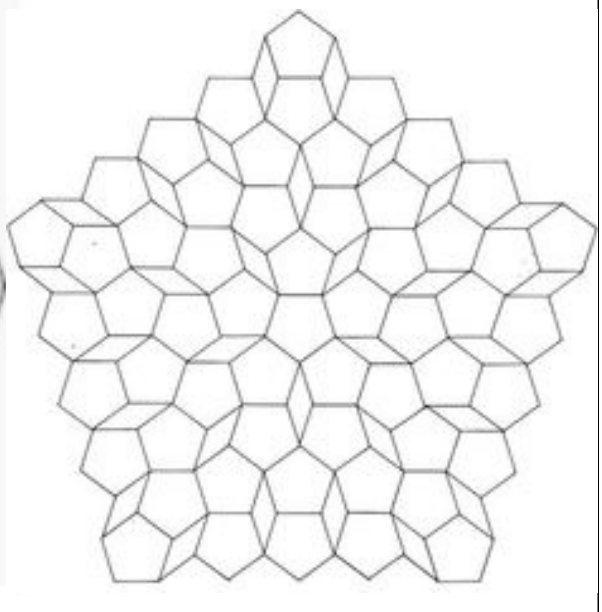




Colour the patterns

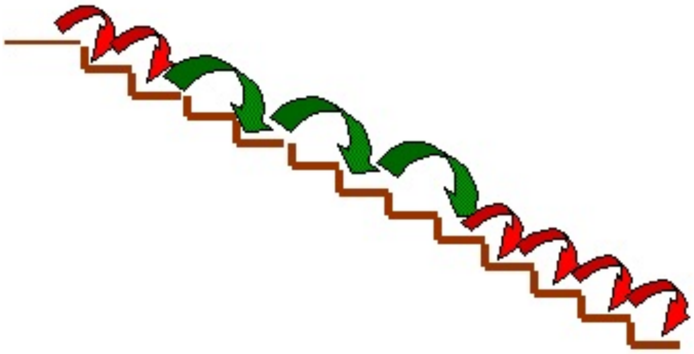








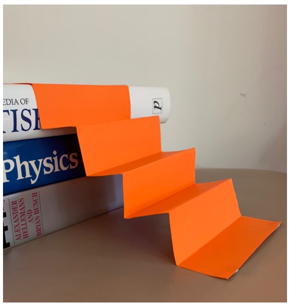
Upper Primary



This staircase has 12 stairs. You can go down the stairs one at a time or two at time.

For example: You could go down 1 stair (as shown in red), then 1 stair again, then 2 stairs (as shown in green), then continue 2, 2, 1, 1, 1, 1 as in the picture.

Suppose there is only one stair. Then there is only one way to go down. It is entered in the table below.



What about 2 stairs? Then 3 stairs…

This orange staircase has only 4 stairs. You could make paper staircases with more stairs.

Can you find the solutions up to 6 stairs and put them in the table? Either do it in your group by actually walking downstairs in all the different possible ways or use a pretend staircase made from folding paper.

Number of stairs	1	2	3	4	5	6
Number of different ways to go down the stairs	1					

Can you see patterns in the numbers in your table?

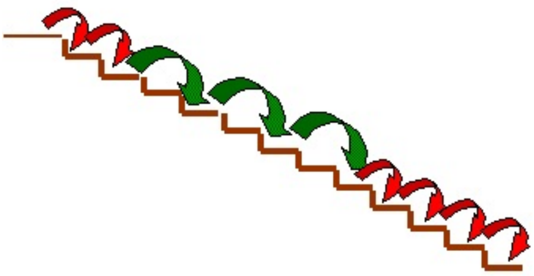
This pattern is connected to THE GOLDEN RATIO, sometimes called THE GOLDEN MEAN, a number approximately 1.62

This was written in a mathematical magazine a few years ago:

The golden mean appears throughout art and nature, including the human body in the ratio of the total height of the average adult male to the height of his navel. The same ratio in the new-born baby is 1:1.

So for children this ratio will be somewhere between 1:1 and 1.62 : 1
Take some measurements and investigate this ratio for yourselves.

Years 7 to 10



Tell the group what the problem is.

Show them the picture.

Talk about how it shows going down in steps of 1 step at a time (shown in red) and 2 at a time (shown in green) which can be written as 1, 1, 2, 2, 2, 1, 1, 1, 1 steps at a time making 12 in all.

Ask learners to suggest other examples for 12 stairs starting with 1, 2, … and 2, 1,… and agree that there must be a large number of different possibilities.

You might even try this if you have stairs in your home.

Then suggest that the group should start with simple cases, with 1 step to go down, then 2 steps, then 3 steps etc and look for a pattern.

Give them a few minutes to work out the number of different ways for n = 1, 2 and 3 then agree the answers so that everyone is helped to understand what is involved.

Then ask the group to work out the answers for 4, 5 and 6 stairs. They could work in pairs if you are a big group and then one pair could check their answers with another pair to see if they have got answers the other pair have not found, and vice versa.

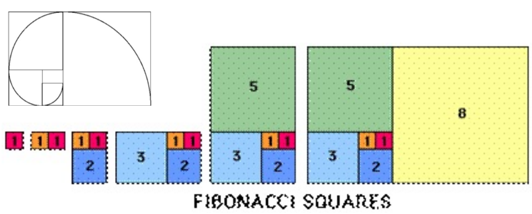
Finally ask the learners to explain their answers for 4, 5 and 6.

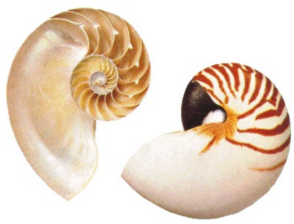
Then write down the sequence of the number for 1, 2, 3, 4, 5 and 6 stairs – that is 1, 2, 3, 5, 8, 13 … and ask the learners to look for a pattern.

When someone spots the pattern they must explain it to the everyone else. Then the group should continue the sequence 1 ,2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233 … to find the answer for 12 stairs which is 233.

This assumes that the pattern of solutions to the One Step Two Steps problem continues from 6 to 12 stairs and that needs to be proved.

Here are some examples showing the Fibonacci sequence in nature and art. If you have access to the internet your learners might do some searches for themselves, to find out about the Golden Ratio.





At the end of the session you could do the Diagnostic Quiz with Year 9 to 13 students.

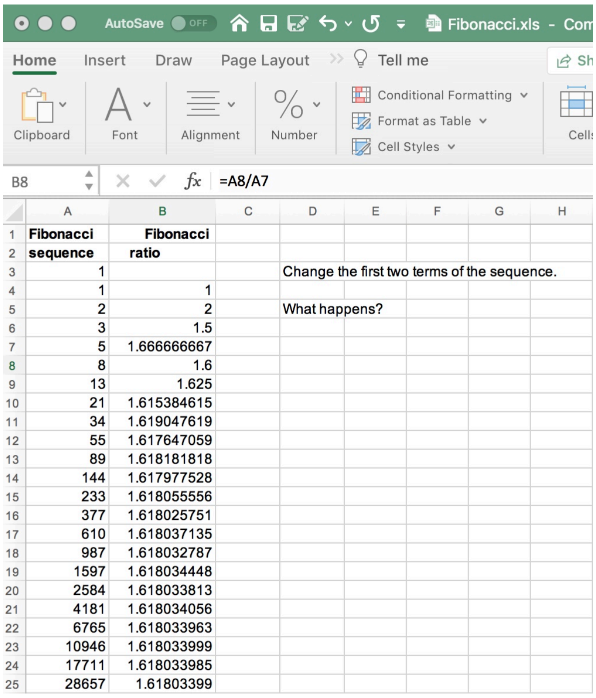
Years 11 The Golden Ratio in an Excel spreadsheet

Each term in a Fibonacci sequence is the sum (total) of the previous terms.

Can you prove this result rigorously?

If we use S_n for the number of ways of going down n stairs then S_{n-1} is the number of ways for (n – 1) stairs and S_{n-2} is the number of ways for (n – 2) stairs.

So the rule to be proved is $\mathbf{S_n = S_{n-1} + S_{n-2}}$



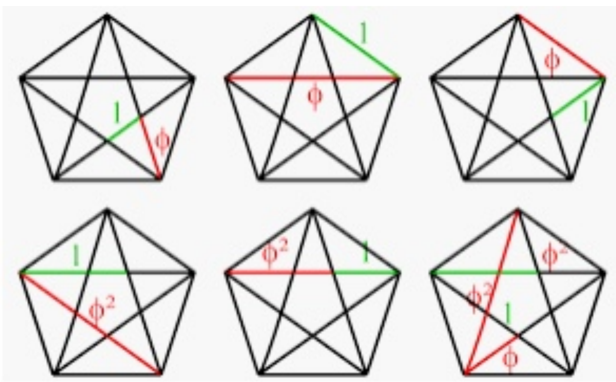
	A	B	C	D
1	Fibonacci	Fibonacci		
2	sequence	ratio		
3	1			Change the first two terms of the sequence.
4	1	1		
5	2	2		What happens?
6	3	1.5		
7	5	1.666666667		
8	8	1.6		
9	13	1.625		
10	21	1.615384615		
11	34	1.619047619		
12	55	1.617647059		
13	89	1.618181818		
14	144	1.617977528		
15	233	1.618055556		
16	377	1.618025751		
17	610	1.618037135		
18	987	1.618032787		
19	1597	1.618034448		
20	2584	1.618033813		
21	4181	1.618034056		
22	6765	1.618033963		
23	10946	1.618033999		
24	17711	1.618033985		
25	28657	1.61803399		

This image shows a spreadsheet set up to give the Fibonacci sequence and the ratio of each term to the term before it.

Can you set up your own spreadsheet like this?

Then change the first two terms of the sequence to discover what happens.

Years 12 and 13



Do the activities for Year 11.

The activities in this collection for young children involved the regular pentagon. You might like to look at them.

In each of these images the ratio of the red length to the green length involves the golden ratio ø. Can you prove these results?

SOLUTION

1 Stairs	1	Total 1 way
2 Stairs	1,1 or 2	Total 2 ways
3 Stairs	1,1,1 or 1,2 or 2,1	Total 3 ways
4 Stairs	1,1,1,1 or 1,1,2 or 1,2,1 or 2,1,1 or 2,2	Total 5 ways
5 Stairs	1,1,1,1,1 or 1,1,1,2 or 1,1,2,1 or 1,2,1,1 or 1,2,2 or 2,2,1 or 2,1,2 or 2,1,1,1	Total 8 ways
6 Stairs	1,1,1,1,1,1 or 1,1,1,1,2 or 1,1,1,2,1 or 1,1,2,1,1 or 1,2,1,1,1 or 1,1,2,2 or 1,2,2,1 or 2,1,1,2 or 2,1,2,1 or 2,2,1,1 or 2,2,2 or 2,1,1,1,1 or 1,2,1,2	Total 13 ways
etc.		

Each term in the sequence of total number of ways is the sum of the previous two terms.

If we use S_n for the number of ways of going down n steps then S_{n-1} is the number of ways for n -1 steps and S_{n-2} is the number of ways for n-2 steps.

So the rule is $S_n = S_{n-1} + S_{n-2}$

The reason is that at the top you have 2 choices

(1) Go down 1 step then there (n – 1) steps to go and you have S_{n-1} ways of going down them or
(2) Go down 2 steps then there (n – 2) steps to go and you have S_{n-2} ways of going down them.

So to find how many ways altogether you need to add all the ways for these two choices so $S_n = S_{n-1} + S_{n-2}$.

So for more steps we use the sequence 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, … giving the answer 233.

These are the Fibonacci numbers starting at 1, 2, … from the Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, …

This links to a lot of mathematics about The Golden Ratio with applications and examples in nature, in art and in geometry.

Key Questions

- Are you sure that you have listed all the possibilities? How do you know?
- Have you listed the possibilities in a special order?
- How about listing the steps taken starting with 1 step, then the steps taken starting with 2 steps.
- Look at any three successive terms, do you notice anything?
- Is there a pattern in the sequence 1, 2, 3, 5, …?
- What do you think the next term might be? Can you make a list of the possibilities and check it?

Why do this activity?

This activity will help learners to develop problem solving skills, in particular to start with simple cases for 1 step, then 2, then 3, then 4 etc. and to look for a pattern in the sequence of answers. Working on this problem also gives learners an experience of the advantage of working systematically.

Learners will meet arithmetic and geometric sequences where the n^{th} term can be written as a formula involving n. Unlike these sequences the terms of the sequence for this activity 1, 2, 3, 5, 8, … do not change in equal steps or with a common ratio. To meet a sequence that is defined quite differently, and one that is found widely in different examples in nature, in art and in geometry, will broaden their knowledge and appreciation of mathematics.

The problem also involves numbers of different arrangements or permutations, a topic which they will meet later in school, but here it is met it in a sufficiently simple way to make it accessible at this level.

Learning objectives

In doing this activity students will have an opportunity to:

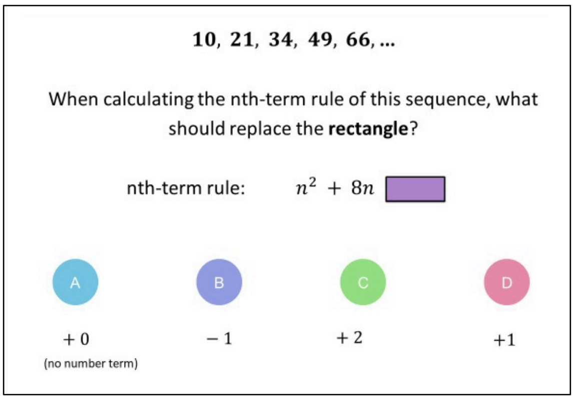
- investigate numeric and geometric patterns looking for relationships between numbers;
- extend a sequence following a pattern;
- develop problem solving skills;
- make and prove a conjecture about the formula for a Fibonacci sequence.

Generic competences

In doing this activity students will have an opportunity to:

- develop problem solving skills;
- make a plan and work systematically to find all possible solutions to a given problem.

DIAGNOSTIC ASSESSMENT FOR YEARS 9 to 13 This can be done before or after the lesson and as a group as described below, or the question can be answered individually.
Show this question and say:
"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 for D".



1. Notice how the learners respond. Ask them to explain why they gave their answer and DO NOT say whether it is right or wrong, simply thank the learner for the answer.
2. It is important for learners to explain the reason for their answer so that, by putting their thinking into words, they develop communication skills and gain a better understanding.
3. With a group, make sure that other learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the learners to vote again for the right answer by putting up 1, 2, 3 or 4 fingers. Look for a change and who gave right and wrong answers.

The correct answer is: +1 - the formula for the sequence is $S_n = n^2 + 8n + 1$
https://diagnosticquestions.com

Follow up

Elephant Dreaming https://aiminghigh.aimssec.ac.za/years-8-12-elephant-dreaming/

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za
Subscribe to the **MATHS TOYS YouTube Channel** https://www.youtube.com/c/mathstoys
Download the whole AIMSSEC collection of resources to use offline with the AIMSSEC App see https://aimssec.app Find the App on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13