AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

LOOPY

$$a_{n+2} = \frac{1 + a_{n+1}}{a_n}$$

The terms of a sequence are given by the equation where each term is found from the two preceding terms.

For example if the first two terms are 2 and 5 then the next term is (1+5)/2=3 and the sequence is 2, 5, 3, 4/5, 3/5, 2, 5, ... etc.

Investigate the sequences you get when you choose your own first two terms. What happens to your sequences?

Make a conjecture about the behaviour of sequences given by this rule.

Can you prove your conjecture using algebra?

HELP

The terms a_n, a_{n+1}, a_{n+2} are three successive terms in the sequence and the subscripts n, n+1 and n+2 denote the nth term and the two terms in the sequence that follow.

So suppose $a_n = 2$ and $a_{n+1} = 5$ then $a_{n+2} = \frac{1+a_{n+1}}{a_n} = \frac{1+5}{2} = 3$ and the next term is $\frac{1+3}{5} = \frac{4}{5}$

Now find the next 3 or 4 terms. What do you notice?

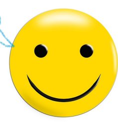
Choose your own first two terms and investigate the sequences that they start.

NEXT

The big challenge here is to use algebra to prove your conjecture. If you substitute in the formula and do the calculations correctly you will always seem to get similar patterns, but there may be exceptions.

Working with algebraic fractions follows all the same rules as working with numeric fractions.

$5^2+8^2=25+64=89$



When you have proved this conjecture you might like to try **Happy Numbers**.

Take any 2-digit number and add the squares of the digits to get the next Happy Number.

For example:

58, 89, 145, 42, 20, 4, 16, 37, 58, 89 … and now the sequence repeats in an 8-cycle

15, 26, 40, 16, 37, 58, … and this sequence starts repeating with the 4th term from 16 onwards.

23, 13, 10, 1, 1, 1, …this sequence is attracted to the fixed point 1. It just repeats 1, 1, … for ever.

What else can you discover about Happy Numbers?