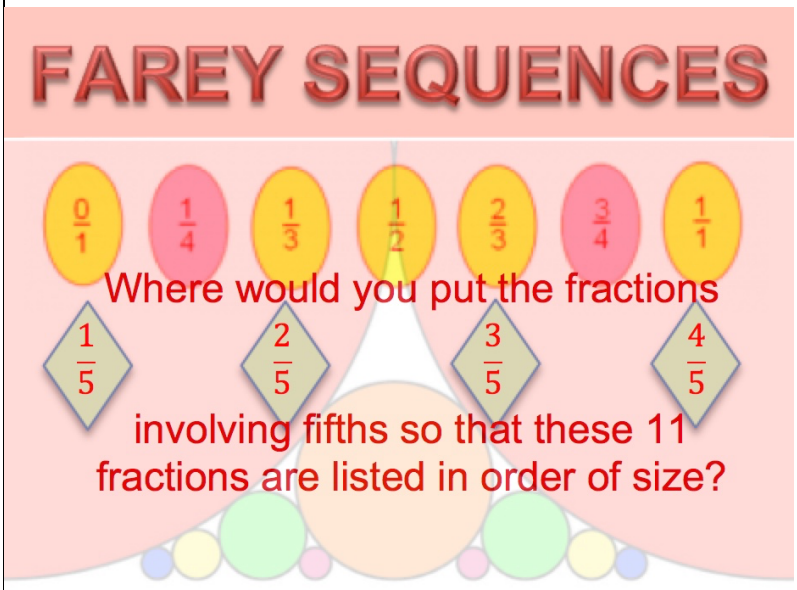
FAREY SEQUENCES

It is not difficult to put a list of decimals in order of size. But what about ordering fractions?

In 1816 John Farey introduced a method for producing sequences of fractions in order of size. Can you discover his method from the following examples?

The third Farey sequence shown here lists in order, in their simplest form, all the fractions between 0 and 1 that have denominators 1, 2 and 3.

The fourth Farey sequence (F4) shown here lists in order, in their simplest form, all the fractions between 0 and 1 that have denominators 1, 2, 3 and 4.

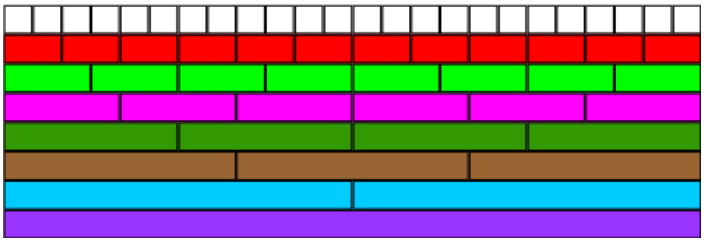


Can you find the fifth Farey sequence?

What about more sequences in this sequence of sequences?

HELP

A fraction wall might help you to match up equivalent fractions.



Notice that in the 4th Farey sequence, F4, $\frac{3}{4}$ is slotted in between $\frac{2}{3}$ and $\frac{1}{1}$

What do you notice about the fractions on either side when you slot in a new fraction?

Choose any three consecutive fractions from a Farey Sequence.

Can you find a way to combine the two outer fractions to make the middle one?

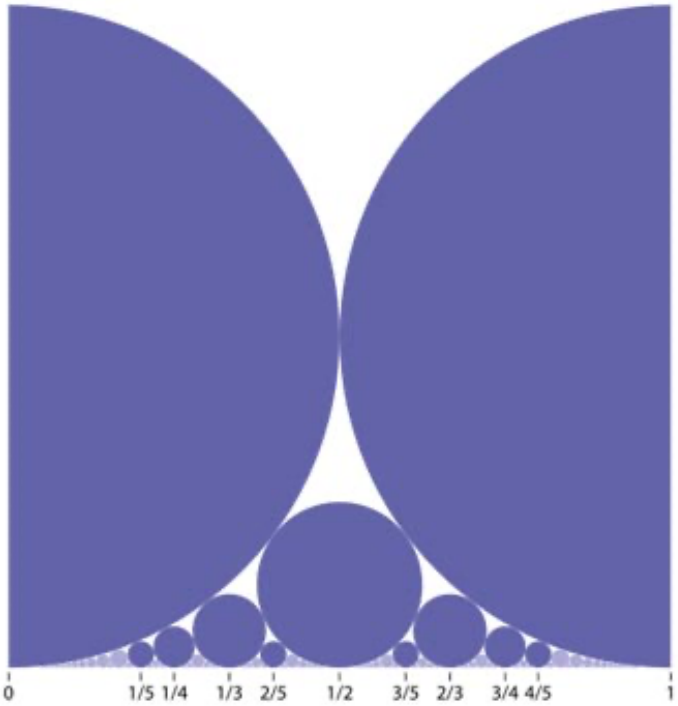
NEXT

Investigate more sequences.

Can you find a Farey sequence with an even number of fractions? Is there more than one?

Why or why not?

Can you find some examples of sequences that have a lot of extra fractions that were not in the sequence before, and others that only have a few extra fractions. Can you explain why this happens?



Mathematics is full of amazing patterns and there is a lot more mathematics involving Farey sequences. What do you notice in this picture?