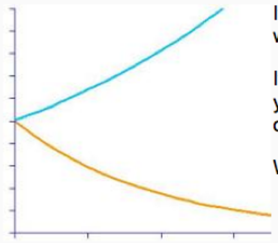
Grades 9 to 12 Dating Made Easier



If a sum invested gains 10% each year how long will it be before it has doubled its value?

If an object depreciates in value by 10% each year how long will it take until only half of the original value remains?

Why aren't these two answers the same?

Is there a rate, used for both gain and depreciation, for which those two answers would actually be the same?

See http://nrich.maths.org/5636

Notes for Teachers

Why do this problem?
Before you give learners a formula for compound interest let them work on this problem and, with a bit of guidance, and some discussion in groups and with the whole class, the learners will discover the formula for themselves and develop a good understanding of the concept of compound interest.
Possible Approach
Start by asking how long it would take for a saving of R1000 to double to R2000 if 10% is added each year and no money is taken out of the savings account. If the learners can work out percentages they will no doubt get R100 interest for the first year. Take a vote in the class. How many think the interest for the second year will be R100 and how many think it will be more? Ask those who think it is more to explain why. When everyone understands why the interest in the second year is R110 ask them to go on to find out how long it takes before the savings amounts to R2000. At this stage it's a good idea to have calculators available for this work. Ask those who finish first if they can find a quick method to do the calculations. When all the learners have worked through the calculation for 8 years, lead the class towards understanding that multiplying by 1.1 is the same as adding 10%.

Then ask the question about how long it takes to halve the amount if you have R1000 and spend 10% of the amount left at the end of each year. Don't suggest multiplying by 0.9 but ask the learners to find the quickest way to do the calculation.

This may take longer than if you just told the class what formula to use but if the teacher handles this well the learners should end up by understanding where the formula comes from and be much more confident about using it and less likely to forget how to use it.

Starting with an amount 1, the year by year amounts are given in this table:

Timeline	Amount gains	Amount depreciates
	10% a year	10% a year
Start	1	1
1	1.1	0.9
2	1.21	0.81
3	1.331	0.729
4	1.4641	0.6561
5	1.61051	0.59049
6	1.771561	0.531441
7	1.9487171	0.4782969
8	2.14358881	

The amount reduces to half the original value in the 7^{th} year and doubles to twice it's value in the 8^{th} year.

For high flyers you can ask the following question and accept any well reasoned answer:
Why does it take longer to reach double the value than it does to halve the value? The answer is that these rates are not inverses of each other.

The following solution is just for teachers and very 'high flyers'.
Suppose and amount A is increased by 10% then it becomes 1.1A.
If this increased amount is decreased by 10% it becomes $0.9 \times 1.1A = 0.99A$ which is less than A.
To get back to A (to make the time for doubling and halving the same) less than 10% would have to be deducted.

Increasing at a rate of r% an amount A doubles when $(1+r)^n A=2A$.
Decreasing by r% it halves when $(1-r)^n A= \frac{1}{2} A$.
If it takes the same time to double as to halve the amount then for some value of n:
$(1+r)^n=2$ and $(1-r)^n= \frac{1}{2}$ for the same value of n.
So $[(1+r)(1-r)]^n = 2 \times \frac{1}{2} = 1$ and hence $(1-r^2)^n=1$ which is only possible for r=0 or n=0.