Grades 6 to 12 Which Hand

Ask a friend to hide a 5c coin in one hand and a 10c coin in the other without showing you which coin is in which hand. Then say to your friend: Multiply the amount of money in your left hand by 2 or 4 (whichever you like) and the amount of money in your right hand by 3 or 5 (whichever you like), then add the results and tell me the total." According to whether the answer given is even or odd you can tell which coin was in which hand. Explain how this trick works.
(Thanks to the Mathematical Digest for this trick).

The 5c coin is an odd amount and the 10c coin an even amount. One operation is multiplying by an odd number, the other multiplying by an even number. The problem uses the idea summarised in the table on the left below.

X	EVEN	ODD
EVEN	EVEN	EVEN
ODD	EVEN	ODD

Multiplying by an even number always gives an even number so the result of the calculation involving the left hand is always even. The only way to get an odd answer is to multiply an odd number by an odd number and it only happens when the 5c coin is in the right hand.

So if the final answer is odd you know the 5c coin is in the right hand. If the final answer is even then the 5c coin must be in the left hand.

Notes for Teaching

Why do this problem?
i. The underlying mathematics is simple but it requires clear reasoning to understand why the trick works.
ii. It is important for learners to know the facts, and to understand why, multiplying an even by an odd number and an odd number by an even gives an even answer whereas multiplying an odd number by an odd number always gives an odd answer.
iii. This activity can be used in introducing the properties of odd and even numbers to younger learners and giving them an experience to help them to remember the concept. Younger learners should be able to explain that multiplying by a multiple of 2 always gives a multiple of 2 and that you have to have a factor of 2 somewhere in the product to get an even answer.
iv. The activity can also be used to re-enforce the idea for older learners who can be asked to use algebra to prove it.
v. The activity gives learners practice in mental arithmetic in an enjoyable context.
vi. Learners may make a conjecture like: "If the answer is odd then the 5c coin is in the right hand". They can test that conjecture and it will always work if they do the arithmetic correctly. To prove the conjecture only requires the learners to check 8 cases. Why are only 8 cases possible and what are they?
vii. This problem is at first mystifying. Later most people have a pleasurable 'AHA' experience when the light dawns and they understand why the trick works.
viii. Learners are motivated to know a trick that they can use to impress their parents and friends.
ix. Explaining why the trick works is a good exercise to develop communication skills.

Possible approach
This could be a warm-up task or lesson starter that is ready on the board so learners get busy as soon as they sit down in class. The challenge need not take up much lesson time, especially for upper secondary school students.

ENCOURAGE LEARNERS TO READ THE PROBLEM AND, IN PAIRS, WORK OUT FOR THEMSELVES WHAT THEY NEED TO DO.

Very often teachers say that learners can do the mathematics in class but do badly in tests. One reason for this could be that learners find it difficult to read the question and work out for themselves what they have to do. Teachers often give too much help by reading the problem and explaining what the learners must do, perhaps using code switching. You might explain to the learners that they need to get used to reading questions for themselves. Give them this list of guidelines for how to read the problem and what to ask themselves.
1. Read it slowly. Read it bit by bit
2. "Do I understand all the words?"
3. If not look them up, ask a friend or ask the teacher.
4. "What am I supposed to do?"
5. How do I know from the answer which hand the 5c coin is in?
6. "Do I know any maths that could help?"

Tell them to take turns being the 'math-magician' and the one who has the coins in each hand. After some time remind the learners that it is a good plan to record their answers.

What to record: You might suggest that learners record the final answer and which coin was in which hand. You might suggest a table starting something like:

Final answer	5c coin in	10c coin in
50c	Left hand	Right hand
65c	Right hand	Left hand

REPORT BACK
If any learners have worked out how the trick works and why, ask two learners to demonstrate the trick but not to tell other people how it works. If no learners have found the reason then demonstrate it yourself .

Give the learners more time to work on the challenge but not necessarily in lesson time. Such challenges can be left for a week or two with repeated encouragement and hints from the teacher so that learners have the incentive to keep trying.

Key question
When are the final answers even?
When are the final answers odd?
Do you notice any pattern in the final answers and which hands the coins are in?
How many different cases are there to consider? What are the choices to make and the possibilities?

Possible extension
Ask the question 'Can you make up a similar trick with another 2 coins (for example with a 20c and a 50c coin)?'

Possible support
If some learners are really struggling you might ask them to complete the following and then to use their own answers to try to explain the trick.

2 x 5 =   3 x 10 =

4 x 5 =   5 x 10 =

3 x 5 =   2 x 10 =

5 x 5 =   4 x 10 =