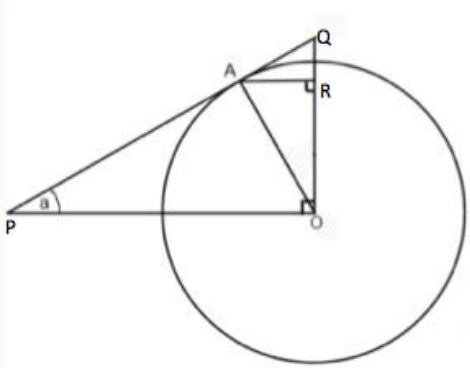


This is a unit circle with a tangent at A.

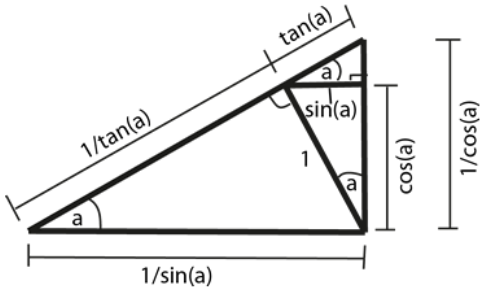
Find the lengths sina, cosa, tana, 1/sina, 1/cosa and 1/tana in the diagram and the areas of the three triangular regions.
See
http://nrich.maths.org/7052

OA = 1
∠OPA=∠AOR=∠QAR=a
AR=sinA
OR=cosa
AQ=tana
OP=1/sina
OQ=1/cosa
AP=1/tana
Area OAP = ½ 1/tana
Area OAQ = ½ tana
Area OPQ = ½ 1/sinacosa

There are many ways to create the solution as it is something like a jigsaw. We used this method

1. Mark the right angles (use the fact the a radius and tangent at a point are at right angles)
2. Make all occurrences of the angle a
3. Mark the unit length

The diagram on the right gives all the lengths.



Notes for teachers

Why use this problem: To train your learners **to think for themselves** you need to give them non-routine problems based only on a small amount of factual knowledge as in this example. The only knowledge required is the definitions of sin, cos and tan in a right angles triangle and the area of a right angked triangle.

Possible approach This example works well as a 1-2-4-more exercise because the learners should be able to get started by themselves. Checking answers and then trying to work out the other answers with a partner and then checking those answers with another pair of learners will help them to check their own work. Finally, the 'more', that is class discussion, will help everyone to see some different methods for getting the solutions and they will be able to check all their own answers.

Key Questions

1. How many right angles can you see?
2. Can you find other angles in the diagram equal to angle a?
3. In a right angles triangle if you know one side and all the angles can you find the other two sides?
4. Once you know all the lengths how do you find the areas?

Possible Extension Find the areas of all the regions in the diagram (including the areas of the sectors). For the solution to this extended problem see: http://nrich.maths.org/7052&part=solution